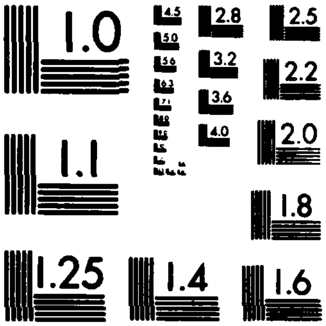
AD-A133 023 LIFE CYCLE COSTING IN A DYNAMIC ENVIRONMENT(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH J A LONG 1983 AFIT/CI/NR-83-26D 1/3

UNCLASSIFIED F/G 5/1 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

UNCLASS

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFIT/CI/NR 83-26D	2. GOVT ACCESSION NO. AD-A133023	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Life Cycle Costing in a Dynamic Environment		5. TYPE OF REPORT & PERIOD COVERED ~~THESIS~~/DISSERTATION
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John Amos Long		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS AFIT STUDENT AT: The Ohio State University		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS AFIT/NR WPAFB OH 45433		12. REPORT DATE 1983
		13. NUMBER OF PAGES 189
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASS
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

APPROVED FOR PUBLIC RELEASE: IAW AFR 190-17

1 SEP 1983

Approved for public release: IAW AFR 190-17.
LYNN E. WOLAVER
Dean for Research and Professional Development
Air Force Institute of Technology (ATC)
Wright-Patterson AFB OH 45433

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

ATTACHED

DTIC ELECTE SEP 1983 E

DTIC FILE COPY

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASS

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

83 09 14 082

LIFE CYCLE COSTING IN A DYNAMIC ENVIRONMENT

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

John Amos Long, A.B., M.A., M.S.

* * * * *

The Ohio State University

1983

Reading Committee:

Gordon M. Clark

Walter C. Giffin

Clarence H. Martin

Approved By

Adviser
Department of Industrial and Systems Engineering

LIFE CYCLE COSTING

IN A

DYNAMIC ENVIRONMENT

By

John Amos Long, Ph.D.

The Ohio State University, 1983

Professor Walter Giffin, Adviser

The consideration of life cycle cost is a major part of the Department of Defense management strategy to control the increasing cost of defense systems. It includes the cost of research and development, production, operating and support, and disposal. Unfortunately, due to a lack of credibility, life cycle costing has not reached its full potential. In an attempt to rectify the situation, this research centers on life cycle costing in a dynamic environment. This examination is from three perspectives: methodology, modeling, and application. The chapter on methodology is a critical examination of Air Force life cycle costing in the acquisition of new aeronautical systems. It contains recommendations for reorganization and revision of current business practices. The chapter on modeling reviews various models and methods for risk analysis including Monte Carlo simulations, additive and

multiplicative moments, sums and products of random variables, and transform techniques. These methods are then directly applied to the problem of operating and support cost estimation. Included is a discussion of candidate probability distributions and suggestions for presentation of the risk analysis. The chapter on application demonstrates the feasibility of using the various models and methods under a realistic scenario for systems acquisition. Therefore, in order to enhance the credibility of life cycle costing, all three aspects (methodology, modeling, and application) are necessary. With its intuitive appeal and following the recommendations and procedures set forth in this research, life cycle cost holds great potential in managing the nation's defense resources.

Key Words: Life Cycle Cost, Risk Analysis

Rank and Service: Major, U.S. Air Force

Pages: 199

ACKNOWLEDGEMENTS

I wish to thank my adviser, Dr. Walter Giffin, and the remainder of my research committee for their guidance, patience, and understanding in completing this task. I also wisk to thank Dr. Darrell Roach for his assistance in preparing for the interviews conducted as a part of this research.

My appreciation is also extended to all those who submitted themselves to the interviews and, in particular, to Mr. Vera Menker for his help in arranging and scheduling the interviews.

I extend my gratitude to my wife, Nancy, and daughters, Amy and Abigail, for their love and support through this ordeal. Above all, I thank and praise the Lord for His presence and grace in times of trial.

VITA

October 27, 1946........	Born - Harrisburg, Pennsylvania
1968....................	A.B., Miami University, Oxford, Ohio
1968....................	Entered Active Duty - United States Air Force
1968-1977...............	Navigator and Radar Navigator, B-52, Wurtsmith AFB, Michigan, Seymour-Johnson AFB, North Carolina
1976....................	M.A., Central Michigan University, Mt. Pleasant, Michigan
1978....................	M.S., Air Force Institute of Technology, Wright-Patterson AFB, Ohio
1979-1980...............	Life Cycle Cost Analyst, Air Force Acquisition Logistics Division, Wright-Patterson AFB, Ohio
1980-1983...............	Student, Department of Industrial and Systems Engineering, The Ohio State University

FIELDS OF STUDY

Major Field: Industrial and Systems Engineering

Studies in Operations Research. Professors Walter C. Giffin, Gordon M. Clark, and Clarence H. Martin

Studies in Logistics Engineering. Professors Walter C. Giffin and Gordon M. Clark

Studies in Applied Statistics and Experimental Design.
Professor John B. Neuhardt

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter I

INTRODUCTION

In these times of economic difficulty and deficit budgets, the high cost of defense systems and rapidly increasing cost of supporting them once they are deployed is of great concern to the Department of Defense (DOD). The need for affordable equipment in terms of both initial cost and support cost becomes more critical as the present budget trends continue. To combat this problem, the application of the life cycle cost (LCC) concept is receiving greater emphasis. The LCC concept was introduced in the DOD in the early 1960's primarily because of increasing concern over the consequences of competitive procurement without regard to total lifetime cost of a weapons system[1]. Today, LCC is a major part of the DOD management strategy to control the increasing cost of defense systems.

---

[1] A system is a separate, identifiable entity for which costs can be accrued and tracked. What may be a system from one perspective may be a subsystem or component part from another. Thus, a system, for example, may be an aircraft, its electrical system, or an avionics component such as an inertial navigation unit.

Prior to the inception of LCC, the federal government customarily sought to buy the least expensive product available [95:1]. Contracts normally were awarded to the lowest bidder. Although there were exceptions, this practice resulted in the acquisition of many weapons systems that were expensive and difficult to maintain. The essential missing element not considered was the cost of ownership, the cost of operating and supporting weapons systems. Quoting from Defense Procurement Circular 115, dated 24 September 1973,

> Since the cost of operating and supporting the system or equipment for its useful life is substantial and, in many cases, greater than the acquisition cost, it is essential that such costs be considered in development and acquisition decisions in order that proper consideration can be given to those systems or equipments that will result in the lowest life cycle cost to the government.

Thus, the objective of life cycle costing is to enable decision makers, during early program phases, to consider all costs of ownership, as well as, those development and acquisition costs which are closest on the fiscal horizon.

Unfortunately, life cycle costing has not reached its full potential. One reason is a lack of credibility in the LCC estimate on the part of managers, decision makers, and, even, cost analysts. Life cycle costing concerns future costs. Consequently, life cycle costing methods and techniques must deal with risk and uncertainty. Because of this risk and uncertainty, they, the users, do not know how much

confidence to place in the LCC estimate. In an attempt to rectify this situation, this research will examine life cycle costing in a dynamic environment. This examination is from three perspectives: methodology, modeling, and application.

1.1 LCC DEFINED

LCC, as defined in Air Force Manual (AFM) 800-11, is "the total cost of an item or system over its full life. It includes the cost of acquisition, ownership (operation, maintenance, support, etc.) and, where applicable, disposal." Acquisition cost includes the cost of research, development, test and evaluation (RDT&E),[2] production[3] or procurement of the end item; and the initial investments required to establish a product support capability (e.g. support equipment, initial spares, technical data, facilities, training, etc.). Ownership cost includes the cost of operation, maintenance, and follow-on logistics support of the end item and its associated support system. The terms "ownership cost" and

[2] Research and development costs are those costs associated with the research, development, test, and evaluation of system hardware and software. It includes the cost for feasibility studies; simulation and modeling; engineering design, development, fabrication, assembly, and test of prototype hardware; initial system evaluation; associated documentation; and test of software.

[3] Production costs are those costs associated with producing the aircraft, initial support equipment, training, technical and management data, initial spares and repair parts, plus many other items required to introduce a new system.

"operating and support (O&S) cost" are synonymous. Thus, the four major cost categories included in the LCC estimate are research and development, production, operating and support, and disposal.

Figure 1 illustrates the need for LCC. Acquisition cost is but the tip of the iceberg. Depending on the system and the length of the life cycle, ownership costs can far exceed the acquisition cost [73:1-1]. "The LCC technique is justified whenever a decision must be made on the acquisition of an asset which will require substantial operating and maintenance costs over its life span" [15:1]. But, life cycle costing is not limited to acquisition decisions alone.

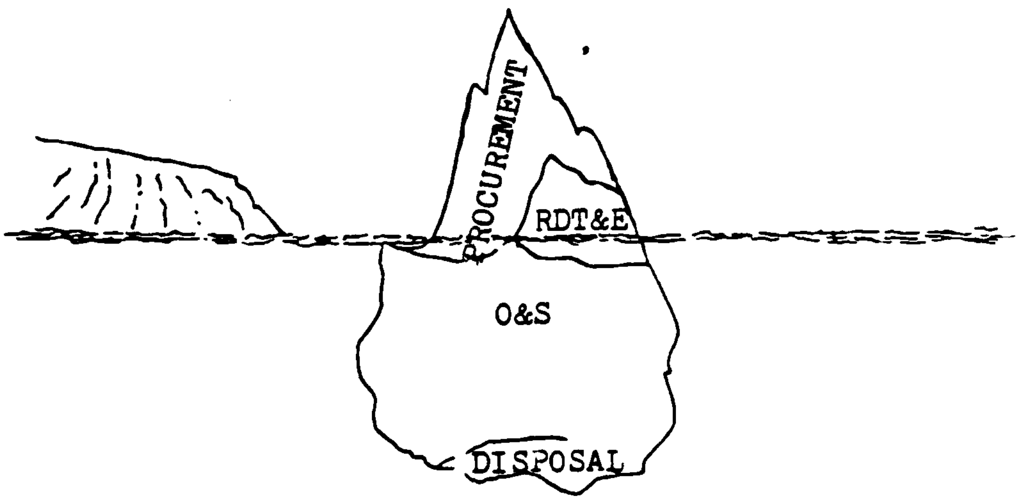


Figure 1: Weapon System Costs

1.2 USES OF LCC INFORMATION

The LCC estimate has many and varied uses. Seldon [95:11-12] lists six primary uses of LCC:

1. Long range planning
2. Comparison of competing programs
3. Comparison of logistics concepts
4. Decisions about the replacement of aging equipment
5. Control over an ongoing program
6. Selection among competing contractors

In addition, May [69:2-3] lists the following uses of LCC estimates:

1. Support of budget estimates
2. Design-to-Cost (DTC)* programs
3. Management reviews

These uses all equate to one common purpose: LCC aids decision makers by supplying information to assist in the decision process. Thus, life cycle costing is really a continuous management process the object of which is to ensure that new acquisitions meet operational needs at the lowest life cycle cost [6:1].

---

* A management concept wherein rigorous cost goals are established during development and the control of system costs (acquisition and operating and support) to those goals is achieved by practical tradeoffs between operational capability, performance, cost, and schedule.

1.3 LCC MANAGEMENT

As a management tool, LCC is supposed to be considererd by all Air Force personnel in making decisions related to the selection, design, development, procurement, production, modification, repair, and use of defense resources. To carry out this mandate, factors which significantly impact LCC must be identified and meaningfull tradeoffs explored. Such tradeoffs involve the selection of design and cost goals, acquisition strategy, sources of goods and services, and support concept. For a new acquisition, the program manager is responsible for LCC management efforts, as well as all other aspects of program management. Life cycle cost management efforts must be tailored to each individual program and include proper documentation of LCC activities, studies, and analyses to support program decisions. The focus of such studies and analyses is the estimate itself. Depending upon the program phase and information available, several techniques are available for arriving at an estimate of total LCC.

1.4 ESTIMATING TECHNIQUES

The three most often used cost estimating techniques in the Air Force are analogy, parametric estimation, and engineering estimation. Analogy is, perhaps, the simplest of the three. The analyst begins by identifying an existing system

that is similar to the system of interest. The cost of the system of interest is then estimated by taking the cost of the existing system and adjusting it to account for differences between the two systems. Although widely used, analogy has several limitations. Analogy places heavy reliance on the opinion of experts to determine the similarities and differences between the two systems. Two experts, given the same information, often have different opinions. Thus, the analysis may not be reproducible, may not be traceable, and may be difficult to document. On the positive side, estimates using analogy are usually fairly easily and quickly done. Analogy is used mainly in the early stages of weapons system development when the least is known about the final end product.

Parametric costing involves the use of a cost estimating relationship (CER). A CER is a mathematical equation or model that relates one or more characteristics of the system to cost. It is a function of one or more independent variables which yields cost as a dependent variable [75:46]. The equation can be simple or complex, linear or non-linear. For example, a CER may be

$$\text{Airframe cost} = \text{Pounds of metal} \times \text{Cost per pound} + \text{Labor hours} \times \text{Cost per hour}$$

or

$$\text{Airframe cost} = \text{Weight}^2 \times \text{Speed}^3.$$

CERs are developed through analysis of past data, often involving regression analysis. CERs are used when system hardware has been defined and physical characteristics are available.

The third estimating approach is the "grass roots" or engineering method, also known as the bottom-up approach. The analyst begins at the lowest level (highest level of detail) and works up adding costs as they occur. This method requires detailed knowledge of the system. The drawback is that intricate detail is needed and, by the time the analyst is able to apply this method, it is usually too late to significantly influence crucial design and support decisions.

Cost estimating models using any or all of these methods generally fall into the broad class of models known as accounting models. Accounting models begin with a cost element structure (CES) which is simply a list of the cost items or categories to be included in the final estimate. All relevent cost categories should be included. The cost elements are then added or "accounted for" in arriving at the total cost. The Air Force approved CES for O&S cost is shown in Table 1 .

Those cost elements which make a significant contribution to the total cost are known as cost drivers; they require special attention from the analyst for it is among these that decision makers will be looking for tradeoffs to reduce

OPERATING AND SUPPORT COST

- UNIT MISSION PERSONNEL
  - Aircrew
    - Military
  - Maintenance
    - Military
    - Civilian
  - Other Unit Personnel
    - Military
    - Civilian
- UNIT LEVEL CONSUMPTION
  - Petroleum, Oil & Lubricants
  - Maintenance Material
  - Training Ordnance
- DEPOT LEVEL MAINTENANCE
  - Airframe Rework
  - Engine Rework
  - Component Repair
  - Support Equipment
  - Software
  - Modifications
  - Other Depot
  - Contracted Unit Level Support
- SUSTAINING INVESTMENT
  - Replenishment Spares
  - Replacement Spt Equip
  - Modification Kits
  - Other Recurring Investment
- INSTALLATION SUPPORT PERSONNEL
  - Base Operating Support
    - Military
    - Civilian
  - Real Property Maintenance
    - Military
    - Civilian
  - Medical
    - Military
    - Civilian
- INDIRECT PERSONNEL SUPPORT
  - Misc Oper and Maint
  - Medical O&M Non-Pay
  - Perm Change of Station
  - Temp Additional Duty Pay
- DEPOT NON-MAINTENANCE
  - General Depot Support
  - Second Dest Transport
- PERSONNEL ACQ & TRAINING
  - Acquisition
  - Individual Training

TABLE 1

O&S Cost Element Structure

cost. This is not to say, however, that other cost elements should be ignored. A previously ignored element may suddenly turn into a cost driver. For example, fuel costs were once insignificant when compared to other operating costs. Now they are quite significant and fuel conservation measures are receiving the highest priority.

There is no specific cutoff point for determining cost drivers, nor does a sudden change alone necessarily produce a cost driver. The selection of cost drivers is at the discretion of the analyst or at the direction of decision makers.

1.5 RISK AND UNCERTAINTY

All aspects of life in the world are subject to risk and uncertainty. Risk and uncertainty are key characteristics of any long range planning and cost estimation. Few, if any, decisions are made under conditions of certainty and without risk. Due to the complexities involved, analysts and decision makers must specifically and explicitly address this risk and uncertainty in performing their assigned tasks. Although the terms risk and uncertainty are often used interchangeably, they are not the same. Risk is the probability that a planned event will not be attained within constraints (cost, schedule, performance) by following a specified course of action [64:18]. Uncertainty is incom-

plete knowledge [64:18]. Fisher [34:202] says, "A risky situation is one in which the outcome is subject to an uncontrollable random event stemming from a known probability distribution. An uncertain situation, on the other hand, is characterized by the fact that the probability distribution of the uncontrollable random event is unknown." Canada [18:252] relaxes these definitions somewhat by concluding that risk is the dispersion of the probability distribution of the element under consideration while uncertainty is a lack of confidence that the probability distribution is correct. It is the task of analysts to try to reduce uncertainty to risk and then to meaningfully convey the risk to decision makers.

1.5.1 Sources of Uncertainty

There are two primary sources of uncertainty affecting LCC estimates. These are environmental uncertainty and cost estimating uncertainty. Environmental uncertainty is the product of unforseen changes in politics, engineering, quantity, support concept, schedule, policy, requirements, use, or life cycle. These environmental changes are outside the control of cost analysts. If these areas are held constant, there is still some uncertainty, cost estimating uncertainty.

Cost estimating uncertainty is more easily addressed by analysts. It stems from an inability to measure cost precisely, inadequacy of applicable data, statistical uncertainty, errors or inconsistencies in the treatment of data, and errors in judgement [57:3-10]. Thus, cost estimating uncertainty has both statistical and subjective aspects. The subjective aspect is introduced in conducting the analysis itself. The assumptions used and the decisions made by analysts in performing the study are a source of subjective variance. The statistical aspect results from the reduction and analysis of historical data and the modeling methods and techniques employed. Because this research is primarily concerned with cost estimating uncertainty, a 'fixed scenario' with respect to environmental uncertainty is assumed. Both the statistical and subjective aspects of cost estimating uncertainty will be addressed.

1.5.2 Capturing Risk and Uncertainty

Analysts cannot eliminate risk and uncertainty from a program. At best, they can present and explain the aspects of risk and uncertainty impacting the program. This is done through risk and sensitivity analysis.

Risk analysis is a procedure for analyzing how randomness affects the total cost. To place a cost estimate in proper perspective, it must be viewed as a random variable. By

definition, a random variable is a numerically valued function defined over the sample space [47:327]. Unfortunately, the application of risk analysis, particularly in the case of O&S costs, seems limited. Authors and analysts, such as Large [61], McNichols [71], and Worm [104], have addressed the problem of risk in hardware cost estimation, but few have examined O&S cost. A notable exception is Dienemann [27].

Uncertainty is addressed through the application of sensitivity analysis. Although often mistakenly used as a substitute for risk analysis, sensitivity analysis is designed to systematically explore the implications of varying assumptions about the future environment and is normally centered on the cost drivers where a range of alternative parameters is investigated. The objective is to identify those parameters whose change will impact the decision at hand. Risk analysis and sensitivity analysis are complementary and, as such, are a vital and necessary part of every cost analysis.

1.6 RESEARCH QUESTION

The research question is 'How do you do life cycle costing in a dynamic environment?'.

This dissertation begins with an examination of the environment in which life cycle costing is done. Problems con-

fronting managers, decision makers, and analysts are addressed. Next, various modeling methods are explored. The primary focus is on analytic methods using the analogy and parametric costing estimating techniques. Then, applications demonstrating these modeling methods and techniques are presented. A goal of the last two phases is to produce an approach to risk analysis which can be easily understood and applied by the analyst in the field. Thus, the thrust of this research is in three main areas: methodology, modeling, and application.

Chapter II

METHODOLOGY

This chapter critically examines the Air Force LCC method and methodology. It begins by looking at the present LCC structure and evolves into a discussion of problems and suggestions for improvement. The material presented is the result of an indepth literature search, personal interviews with key Air Force and DOD life cycle costing personnel,[5] and the observations and experience of the author. This chapter focuses on the acquisition and, in particular, support of new aeronautical systems. These systems consume a major part of the Air Force acquisition dollars and O&S is typically the LCC driver.

---

[5] Appendix A contains a listing of those interviewed. To ensure a candid response from each, it was agreed that their names would not appear in the text without expressed permission.

2.1 THE AIR FORCE ACQUISITION PROCESS

Before discussing the LCC structure, a brief review of the Air Force acquisition process is in order. This process begins with a threat and a need to counter that threat. Identification of the threat and subsequent need may come from within the Air Force or external to it. Once threat and need have been identified, an acquisition program begins. Major systems acquisition is normally divided into the following phases: concept exploration, demonstration and validation, full-scale development, and production and deployment. The emphasis is on decentralized management tailored to the individual programs.

The concept exploration (conceptual) phase begins with the identified need and a more detailed requirements definition. The Air Force prepares a justification of major systems new starts (JMSNS) and requests funds. The Secretary of Defense issues appropriate program guidance and authorizes the service to proceed. Studies, tests, and analyses of experimentally developed hardware establish the technical, military, and economic bases for the program [73:2-2].

The first major Secretary of Defense decision occurs after concept exploration and signals entry into the demonstration and validation phase. This is known as Milestone I. During the demonstration and validation phase, program performance, cost, and schedule are validated and refined

through more extensive analysis, hardware development, and prototyping.

Following the demonstration and validation phase, program approval to proceed with full-scale development is sought. This is the second and last decision for the Secretary of Defense and is known as Milestone II. During this phase, the system, including support items and equipment, is designed, fabricated, and tested. It is during this phase that full-scale prototypes are built.

The production decision, made by the Air Force, is known as Milestone III. Production continues until the last unit produced is accepted as operational. Deployment overlaps production, beginning with acceptance of the first operational unit and continuing until deactivation or phase out of the system.

Three Air Force organizations are deeply involved in the process. These are Air Force Systems Command (AFSC), Air Force Logistics Command (AFLC), and the operating or using command. All initial program phases are under the auspices of AFSC, which is tasked with developing and procuring new weapons systems. AFLC and the using command assume supporting roles. At some predetermined point during deployment, program management responsibility transfer (PMRT) occurs. At that time, management, engineering, funding, and procurement responsibility transfers from AFSC to AFLC, which is

then concerned with the logistical support of the system through the remainder of its useful life. AFLC, however, assumes a supporting role with respect to the operating or using command. Examples of operating or using commands are Strategic Air Command (SAC), Tactical Air Command (TAC), and Military Airlift Command (MAC). Thus, one organization develops and purchases new systems (AFSC), another supports them (AFLC), and a third uses them (operating commands).

2.2 THE AIR FORCE LCC STRUCTURE

The Air Force LCC structure must be addressed from two aspects, directives and organizations. The directives establish the requirement and authority for LCC functions and the organizations administer and carry out those directives. Sometimes, however, what an organization is directed to do is not necessarily what it actually does.

2.2.1 The Directives

The basis of the requirement for life cycle costing is contained in the following DOD documents:

1. DODD[6] 5000.1 - Major Systems Acquisition
2. DODI[7] 5000.2 - Major Systems Acquisition Procedures

---

[6] DODD - Department of Defense Directive

[7] DODI - Department of Defense Instruction

3. DODD 5000.4 - Office of the Secretary of Defense, Cost Analysis Improvement Group

4. DODD 5000.28 - Design-to-Cost

The first two, DODD 5000.1 and DODI 5000.2, concern weapons system acquisition. Among other things, they direct the program manager* (PM) to establish and present LCC estimates and goals to the Defense Systems Acquisition Review Council (DSARC). The DSARC is the top level DOD corporate body for system acquisition and provides advice and assistance to the Secretary of Defense regarding acquisition decisions. DODD 5000.4 provides a permanent charter for the Office of the Secretary of Defense (OSD) Cost Analysis Improvement Group (CAIG) and establishes this group as an advisory body to the DSARC on matters relating to cost. As such, the CAIG is the final evaluator of DSARC cost analyses and, thereby, establishes the standards for such analyses. DODD 5000.28 defines the Design-to-Cost (DTC) management process establishing cost (life cycle cost) as a parameter equal in importance to system performance and program schedule. These DOD documents, in turn, establish the need for complementary Air Force documents.

---

* AFSC assigns each new systems acquisition project to a program office. The program manager is the individual within that office who is responsible for the acquisition program management until PMRT.

Within the Air Force, Air Force Regulation (AFR) 800-11, <u>Life Cycle Cost Management Program</u>, states policies, explains procedures, and assigns responsibilites for implementing LCC management concepts and implements DODD 5000.28 for the Air Force. As such, it is the premier Air Force document on the subject. Complementing this regulation is AFR 800-11/AFSC/AFLC Supplement 1 which further defines the roles of AFSC and AFLC in life cycle costing.

Without exception, those interviewed agreed that these LCC policy and requirement documents are clear, concise, and adequate. Some interviewees did feel, however, that instructions on how to actually do a cost analysis were lacking.

2.2.2 <u>The Organizations</u>

Life cycle costing is a function which cuts across numerous and diverse agencies and organizations which are, at times, loosely and informally connected.

Any discussion of LCC organizations must begin with the previously mentioned OSD/CAIG. This group establishes criteria, standards, and procedures concerning the preparation and presentation of cost estimates to the DSARC, and, in turn, the Secretary of Defense. Therefore, it is the ultimate authority with respect to LCC analysis for the entire DOD.

Within the Air Force, Headquarters, United States Air Force, Deputy Chief of Staff for Logistics and Engineering, Directorate of Maintenance and Supply, Acquisition and Communications Group (HQ USAF/LEYE) is the office of primary responsibility (OPR) for LCC management and Headquarters, United States Air Force, Comptroller of the Air Force, Directorate of Air Force Cost and Management Analysis, Cost Analysis Division (HQ USAF/ACMC) is OPR for the analysis aspects of LCC. Here begins a dual line of functionalism, management and analysis, that permeates throughout the Air Force LCC functional structure. The center, unifying element of the dual line of functionalism is the cost estimate itself. Management uses the estimate for decision making, and analysis is required to produce the estimate. The danger in such an organizational climate is that agencies tend to operate independently, particularly in day to day operations. This independence can lead to contradiction and duplication of effort unless communication is maintained. There is no one line of authority to direct, coordinate, and mediate the actions of these two organizations.

At the next level of command, AFSC has designated Headquarters, Air Force Systems Command, Deputy for Acquisition Logistics, Directorate for Program Readiness and Evaluation, Program Evaluation Division (HQ AFSC/ALPA) as OPR for LCC management; while Headquarters, Air Force Systems Command,

Deputy Chief of Staff, Comptroller, Directorate of Cost and Management Analysis, Cost Analysis Division (HQ AFSC/ACCE) is responsible for LCC analysis. The same general organizational pattern is evident in AFLC where Headquarters, Air Force Logistics Command, Deputy Chief of Staff, Acquisition Logistics, Directorate of Acquisition Plans and Analysis (HQ AFLC/AQP) is OPR for LCC management and Headquarters, Air Force Logistics Command, Deputy Chief of Staff, Comptroller, Directorate of Cost and Management Analysis (HQ AFLC/ACM) is responsible for LCC analysis.

These two lines of functionalism finally merge at the division level. On the Systems Command side, Aeronautical Systems Division[9] Comptroller, Directorate of Cost Analysis, Life Cycle Cost Management Division (ASD/ACCL) is the focal point for LCC management and analysis. For Logistics Command, the Air Force Acquisition Logistics Division[10] Deputy for Acquisition Plans and Analysis, Directorate of Concepts and Analysis (AFALD/XRS) is the focal point for both the management and analysis functions. These two divisions, Aeronautical Systems Division and the Air Force Acquisition Logistics Division, are formally tied together through the joint ASD/AFALD LCC Advisory Group. This group serves as

---

[9] Aeronautical Systems Division (ASD) is located at Wright-Patterson AFB, Ohio.

[10] Air Force Acquisition Logistics Division (AFALD) is located at Wright-Patterson AFB, Ohio.

consultant to the ASD program offices on matters relating to life cycle cost.

Within the ASD program offices, the responsibility for LCC implementation resides with the program manager. This responsibility is then delegated to one of several offices. In some programs, it rests in the logistics area under the auspices of the Deputy Program Manager for Logistics (DPML); in others it may be in the program control area or an accounting organization.

Although the names of these various organizations may be long and awkward, the intent is to show the diversity, complexity, and functional nature of the Air Force life cycle cost management and analysis structure. The relationship of these organizations is illustrated in Figure 2. While these organizations exemplify the multi-functional, multi-discipline nature of LCC, they also contribute to some confusion and lack of consistent emphasis within and among various acquisition programs with respect to LCC.

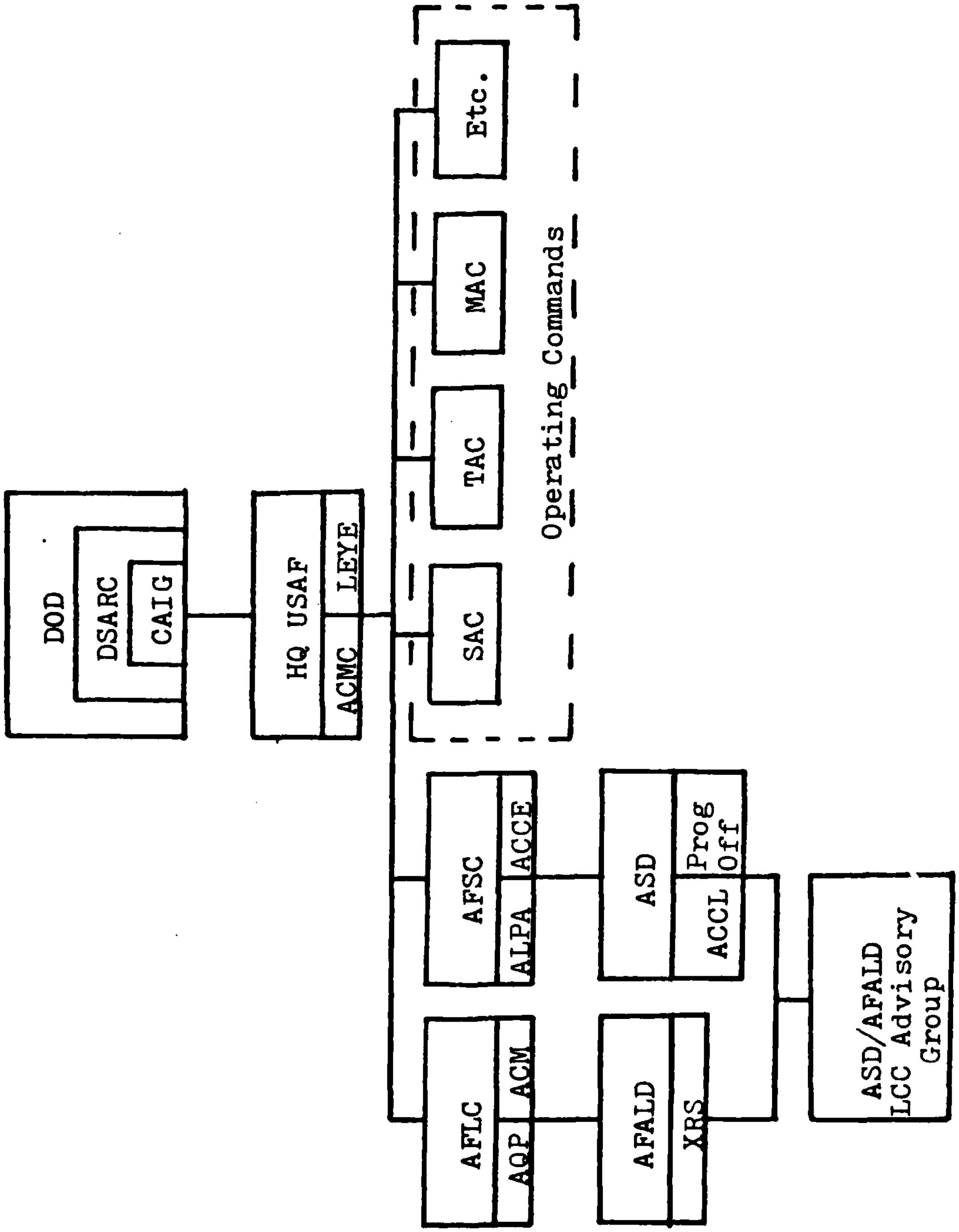


Figure 2: Aircraft Sys Acq LCC Organizational Chart

2.3 THE CREDIBILITY GAP

Without exception, all interviewees agreed that life cycle costing has a credibility problem. Although the causes and reasons varied, this one basic tenet is held by all. Credibility is, then, the key to greater acceptance of life cycle costing as a decision tool.

To be truly effective and credible, life cycle cost must be considered in every decision related to the acquisition of new weapons systems. It cannot be the responsibility of just one person nor can it be the concern of just one group. Anyone concerned with the acquisition process must be keenly aware of the impact of decisions on LCC. In a word, LCC management must be institutionalized.

In particular, it is the early basic decisions made in the life of a program that have the greatest impact on total LCC. The impact of early decisions is illustrated in Figure 3 [16:36]. This figure shows that over seventy percent of the life cycle cost of a system is determined early in the life cycle prior to the concept validation phase approval. By the time production begins, ninety-five percent of the LCC is determined. The remaining five percent is determined during deployment and reflects such things as modifications.

But, it is not only the major decisions that are important. The routine, day to day decisions can cumulatively have a great impact on LCC. As one interviewee put it, "I

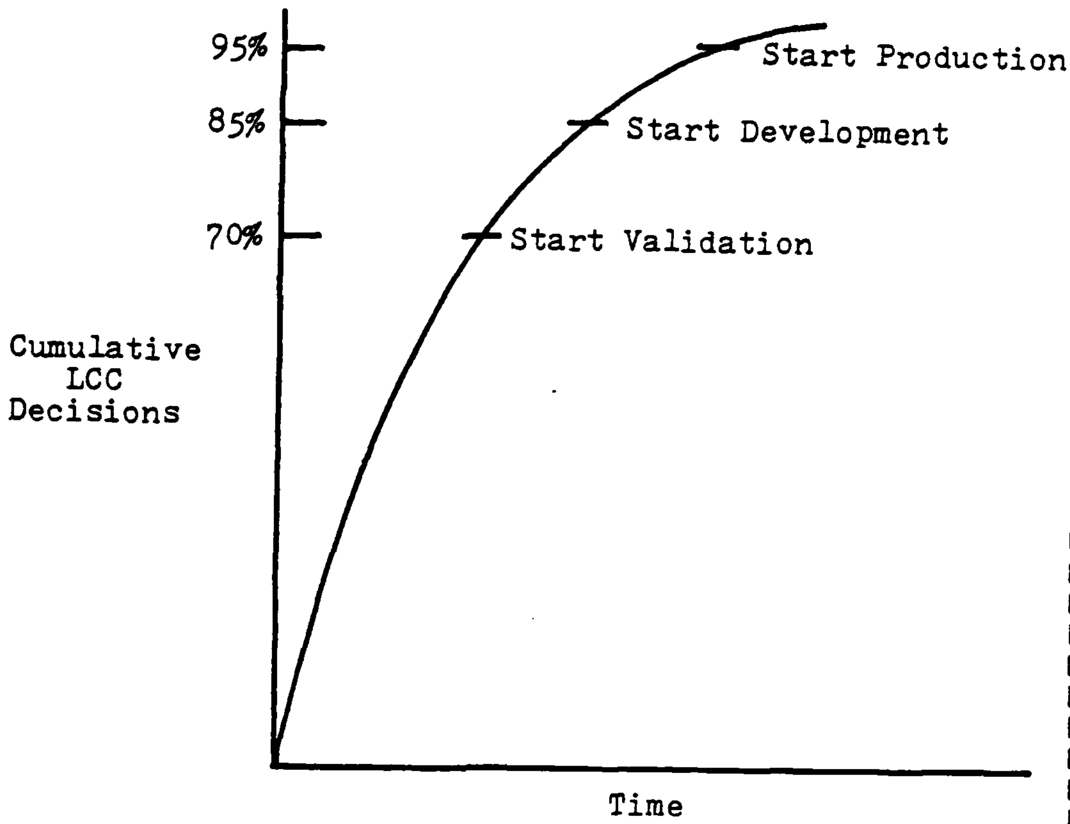


Figure 3: Effect of Program Decisions on Total LCC

would like to see a hand held calculator with LCC model on every engineer's desk." This LCC awareness does not only apply to engineers, but to the entire acquisition and support communities as well.

2.3.1 The Estimate in Perspective

Life cycle cost estimates tend to be confused with budget estimates. Actually, life cycle costing and budgeting are two separate activities done for two separate purposes. Budgeting involves the allocation of monies to be spent over a relatively short time horizon. A LCC estimate can be used in developing and supporting a budget estimate, but it does not include all the items of cost normally associated with budget estimates. If an LCC estimate is not a budget estimate, then what is it?

The LCC estimate is a figure of merit, an aid in the decision making process. It is perhaps unfortunate that the LCC estimate is expressed in terms of dollars. Another unit or, perhaps, an index may be more appropriate. Rather than focusing on the estimate, the focus should be on the decision. Does the cost estimate allow the desision maker to make a more informed decision? Does the LCC figure of merit allow the decision maker to distinguish between or among alternatives? These are the real questions.

Many of the program management decisions where life cycle costs are an important consideration have the characteristics of a classical investment problem. These problems concern whether or not the Air Force should 'invest' additional dollars in the acquisition of a system with increased reliability and maintainability in order to reduce the recurring

costs of ownership [54:3-4]. This is a common capital budgeting problem.

Capital budgeting is the making of long term planning decisions for investments and their financing. A military decision maker, in deciding between or among competing alternatives, is, in effect, making a capital budgeting decision. When the production decision is made, the decision maker is accepting responsibility for the support of that system throughout its useful life. Thus, according to Brown, life cycle costing in this context is not different from capital budgeting but is the application of capital budgeting to nonrevenue-producing projects [15:12]. The objective is to maximize benefits and minimize costs. Therefore, one of the fundamental aspects of such a problem is to be able to state the potential benefits and, in the post-decision environment, to initiate actions to ensure that the benefits are realized. One must thoroughly understand this cause and effect relationship of investment and benefit to make such investment decisions meaningful.

Incredibly, there is no mechanism to ensure that the potential benefits of life cycle costing are actually realized in Air Force programs. This is due, in part, to the Air Force organization. Just as different commands are responsible for the procurement and support of new systems, those same commands are responsible for the financial planning and

fiscal expenditures for the various cost elements which make up the LCC estimate. Additionally, the potential savings are not realized in the same time period as the investment. "If there is never any follow through to insure that benefits are in fact accrued, investment analysis lacks credibility." [54:4]

Added to the problem of follow-up, no organization at Air Force Headquarters monitors the day to day implementation of LCC management. Implementation is loosely monitored through the requirement to include LCC information in various reports and briefings. But this does not ensure that the consideration of LCC is an integral part of the routine decision process within the program office.

The underlying problem that must be addressed, if the life cycle cost management program is to be successful, is to enhance credibility. Credibility must be established by ensuring that the life cycle cost information is relevant to the decision problem, is available to support the timely evaluation of alternatives, and the assurance exists that the actions necessary to achieve the perceived benefits are in fact realized. This requires a means to track and evaluate the effectiveness of the management actions which were initiated.

If life cycle cost management is to become institutionalized in the Air Force, a management system and associated

operating procedures should be established. This system must be responsive to the internal needs of the program manager and provide visilility outside the program office. Such a management program must be judged on its ability to influence the decision process and on the extent to which benefits are realized.

2.3.2 Program Uncertainty

Program uncertainty is the biggest contributor to LCC incredibility. The key to program uncertainty is program stability. The Air Force can do a number of things to improve program stability. First, requirements for new systems must be clearly and explicitly defined early in the program. Then, once the requirements are defined, AFSC, AFLC, and operating command must agree not to unilaterally change those requirements. If change is necessary, all three must agree to the change. If a change or cumulative changes are required that affect the program LCC by a predetermined amount, say twenty percent, then the basic requirements and need for the system should be reexamined and justified. Such a procedure would eliminate some of the many changes experienced in acquisition programs. This is particularly important because once a program is approved, it is difficult to eliminate it. Also, the further along in the acquisition process, the more difficult program elimination becomes.

However, the Air Force is not in full control of weapons system acquisition decisions. Congress, through the budget process, has the ultimate power and authority over all new weapons system acquisitions. A new budget must be approved each year which makes the effective planning horizon one year. Since it takes several years from conception to production of a new system, there is a continual need to justify the program. This detracts from much needed long range planning.

A new Congress convenes every two years. If the budget cycle were lengthened to two years, each Congress would then be required to go through the budget cycle but once, allowing more time to manage that which has been budgeted and approved. This is particularly appealing when one considers that as of February, 1983 a budget for fiscal year (FY) 1983 has still not been approved and the President's budget for FY84 has already been submitted to Congress. Furthermore, if the budget cycle can be lengthened to two years, then why not four years? Each President would then be required to submit but one budget proposal. Again this would allow more time for management activities. If the budget cycle can be stretched to two or four years, why not budget for an entire program phase? Multi-year budgeting and, in conjunction, multi-year contracting would greatly remedy the stability problem. Once approved, as long as a program remained with-

in budget, no further action would be necessary. If the budget constraint were breached, Congress would then be forced to take some action, thus establishing a management by exception philosophy of business. Therefore, a degree of program stability would be achieved and the effective planning horizon lengthened.

Program stability is not just a governmental concern. Business and industry must also be involved. Contractors must be encouraged to deliver systems as specified, on schedule, and within budget. This calls for some special provisions. The government does not conduct business like a private concern. There is a strong feeling within the Air Force procurement community that the government should not be responsible for the demise of a contractor and, in the worst case, the contractor should break even. Thus, there is a reluctance to force the contractors to assume full risk on a project. The government also seems to shoulder a moral obligation due to the many changes made in most programs. Yet the government expects full value on its purchases. Recent efforts for improvement in this area include the use of firm fixed price contracts and an assortment of warranties and guarantees. The real solution, however, is in program stability and improved business practices on the part of both government and industry. In doing so, contractors should be forced to bear responsibility for cost overruns [51:29].

Even if the above recommendations were adopted, some uncertainty and instability would certainly remain. Not even Congress has the reins on the forces of nature. Congress, for instance, cannot control the world price of oil and other raw materials. However, this does not mean that steps should not be taken to control that which is within one's power to control and stabilize that which can be effectively stabilized.

2.3.3 The Program Manager

If stability is the key to program uncertainty, then program managers are the key to LCC management implementation. With program stability somewhat assured, they would be free to do more effective long range planning. As it is now, they must continually justify their programs and manage short term crises. Such short sightedness can lead to suboptimal planning and short term decision making. Further, program managers are not properly motivated with respect to life cycle cost management. The consideration and effective use of LCC in managing is not an integral part of their effectiveness reports. Program managers are evaluated mostly on near term performance which is normally defined to include only the acquisition phase. It is difficult to justify higher initial research and production costs in order to realize uncertain future savings in O&S costs. Supporting this reluc-

tance, the Air Force is so organized that one command (AFSC) procures new systems and another command (AFLC) is responsible for supporting them. Thus, no single individual or command is responsible for a system over its entire life cycle. It should be noted that program managers work for Air Force Systems Command.

This organizational structure also puts the procuring command (AFSC) into the advocate role. Advocacy should be the responsibility of the user. After all, it is the user who best understands the need and solution to that need. Thus, the program manager should work for the using command and both AFSC and AFLC should assume support roles. In this way, program managers can monitor, manage, and control the program not only through the research and production phases, but through the deployment and operations phases as well.

2.3.4 The People Problem

Several interviewees regard the lack of trained, experienced analysts as a problem within the LCC community. This concern is particularly true in O&S costing and is held by both supervisory and non-supervisory personnel alike. Compounding the problem, analysts are not only difficult to acquire, but also difficult to keep. Many analysts work directly with business and industry where the lure of higher salaries and fringe benefits is quite strong. As a consequence, many

of the government's best analysts abandon public service leaving numerous projects in the hands of inexperienced, junior analysts.

Due to the shortage, many projects are assigned to a single analyst with little or no technical assistance. Thus, the typical analyst is not only inexperienced, but expected to be an expert in everything from logistics to engineering to economic analysis. No LCC estimate should be the product of one person's labor. Rather it should represent the efforts of a team skilled in logistics, economics, business, operations, and cost estimating. The task of the cost analyst would then be to coordinate the team effort and produce the final estimate.

2.3.5 The Data Problem

Data is a problem for every analyst in every analysis. There is either too much or too little; it is in the wrong form or format; or its accuracy is questionable. This is true of LCC analysis, but there are some special concerns and problems with regard to Air Force life cycle costing. The remarks in this section are primarily directed at O&S cost data.

First, there are over 140 separate automated and manual data collection banks and systems applying to O&S costs in the Air Force [4:7-34]. Many are old, well established sys-

tems, but are not well documented. None were created for the sole purpose of LCC analysis. However, the biggest problem for analysts is in sorting through this maze of output products to find the information that is needed. For the data to be useful, the analyst must know how it is gathered, what is included or excluded, and what assumptions are used. With such a proliferation of data sources, this can a monumental task.

This problem has been somewhat eased by the Visibility and Management of Operating and Support Cost (VAMOSC) program. VAMOSC is to [91:2]:

1. Develop weapons system O&S cost visibility
2. Develop component level cost visibility
3. Standardize O&S cost terminology and definition
4. Institutionalize the O&S cost system

Using existing data bases, VAMOSC collects and processes raw data producing an output of yearly O&S cost by weapons system in the CAIG approved CES. Unfortunately, the first output did not appear until 1982; thus the number of data points available for analysis are severely limited. In time, however, VAMOSC should evolve into a useful O&S cost analysis tool.

As useful as VAMOSC may prove to be, it does not solve all the data problems. Many of the data collection banks and systems are designed for financial accounting, not cost

accounting, applications. Thus, they are being used for purposes for which they were not intended. This use leads to problems of interpretation, interpolation, and extrapolation. Many categories of historical O&S costs are actually derived values because large portions of the DOD operating and maintenance budget are not identified or apportioned to individual weapons systems or mission design series (MDS) [69:5-1]. Therefore, to provide costs by weapons system, various allocation techniques have been instituted. The quality and reliability of such derived data is then dependent upon the allocation techniques and assumptions applied and how closely they correspond to the actual costs. For example, it is commom for the Strategic Air Command to colocate B-52s and KC-135s at the same base, and it is not uncommon for maintenance technicians to service both aircraft. With the maintenance data collection system accounting for technicians' time, one problem is what basis to use in allocating the idle time between work assignments.

Also, when an item is sent to the depot for repair, it is processed according to its National Stock Number (NSN).[11] Information as to the MDS or base from which the item came is lost. In order to provide cost by MDS, some allocation technique must be applied. In doing so, valuable information is ignored. For instance, one MDS may have more depot

[11] A thirteen digit numeric code assigned to separate hardware items bought by the government.

returns than another on a common item. Allocation may then be made on the number of returns, but the MDS with the fewer returns may actually incur higher cost per repair and, consequently, higher total cost. This lack of traceability also precludes adequate failure modes analysis. Such information is invaluable when evaluating product modifications and improvements, one of the primary uses of LCC.

In spite of the problems with these data collection systems, there is a strong reluctance to change them. They were not intended to be used in weapons system O&S cost analysis. For the purpose they were intended, to serve as financial accounting tracking mechanisms, they do a credible job. Also, changes must be made with care and only after due consideration is given to all the ramifications of the changes because changing data systems could invalidate previously collected data.

2.3.6 <u>The Modeling Problem</u>

Proper and consistent modeling is a continuing problem in analysis. With respect to life cycle costing, as new data becomes available, old CERs must be updated. Models must be tailored to the application and tradeoffs made using comparable cost figures. With such emphasis, there is a tendency on the part of the analyst to become enamored with the model. The model is regarded as an end and not as a means to

an end. When this happens, the analyst must step back from the model and carefully examine the inputs, outputs, and assumptions for reasonableness, consistency, and accuracy. Far too often the output is presented without this critical examination. Then, when a flaw is discovered, it is embarrasing to the analyst and impacts the credibility of the study and life cycle costing in general.

In spite of the drawbacks, models do afford a convenient and orderly way to compute and present cost information. Most cost analyses are accomplished as a combination of the three estimating techniques (parametric, analogy, and engineering) within the framework of a given CES. As a program progresses from the conceptual phase through the demonstration/validation, full scale engineering development, and production phases to the operations phase, there is normally a change in which estimating technique predominates the analysis [69:3-8]. This is shown in Figure 4 .

As the technique changes, so does the model or modeling approach. The problem is that there is no consistent set of models employing the various estimating techniques under a common CES. Each model has its own CES and, therefore, may or may not include the same costs as another. For this reason, outputs between or among models are not comparable.

Therefore, a modular model, using the CAIG approved CES for O&S costs, should be developed. Such a model would al-

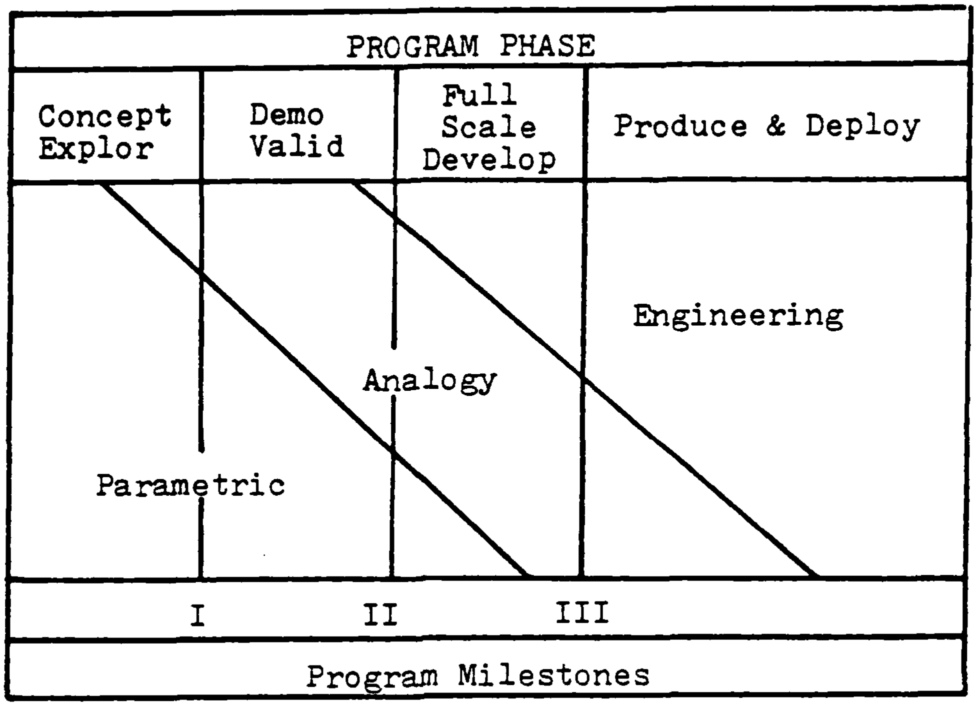


Figure 4: Estimating Technique Applications

low analysts to apply the most appropriate estimating technique to individual cost elements and to change estimating techniques as program phase and available information permit. The result would be a flexible, consistent, dynamic model not only tailored to the application, but also to the program phase and estimating technique.

2.4 THE ROLE OF RISK ANALYSIS

Normally, decision makers are presented with only a point or 'most likely' cost estimate, with no indication as to the risk (variability) in that estimate. For example, Figure 5 shows the relative cost of two systems, A and B. Using cost as the evaluation criterion, and with all other factors being equal, decision makers would choose System A, as it offers the lower LCC. But, point estimates can be misleading and can lead to a worse decision than had no estimate at all been used.

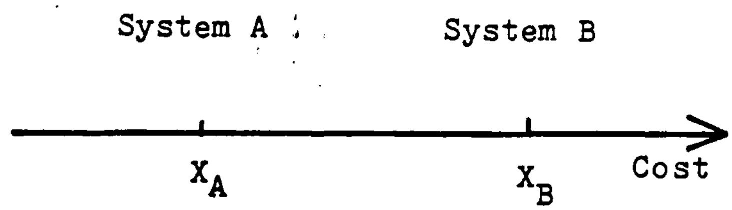


Figure 5: LCC Decision Making

To site an example from Dienemann [27:2-4], Figure 6 shows four cases in which estimates are expressed as prob-

ability distributions to reflect the actual, though perhaps, unmeasurable, uncertainty surrounding each estimate. In Case I, as in Figure 5 , decision makers are faced with no real decision problem because all possible costs of System A are lower than System B; using the point estimate would not affect the decision. The situation in Case II is slightly different in that there is some probability that the actual cost of System A will be higher than System B. If this probability is not large, the decision makers would still select A. However, when the overlap is significant, the point estimate would no longer provide a valid datum for system selection. In the third case, both point estimates are the same, but the cost distribution for B has a larger range or variance. Here decision makers preference toward risk must enter the decision process. If they prefer to minimize risk, they will select A. Case IV is a more complicated situation where the expected cost of System B is lower, but much less certain than A. In this case, if decision makers were to use only a point estimate, they could easily make a wrong or undesirable decision. The application of risk analysis would give much needed visibility into such a decision problem.

In conducting the interviews, however, the application and presentation of risk analysis was met with mixed feeling. Most of those interviewed stated that decision makers

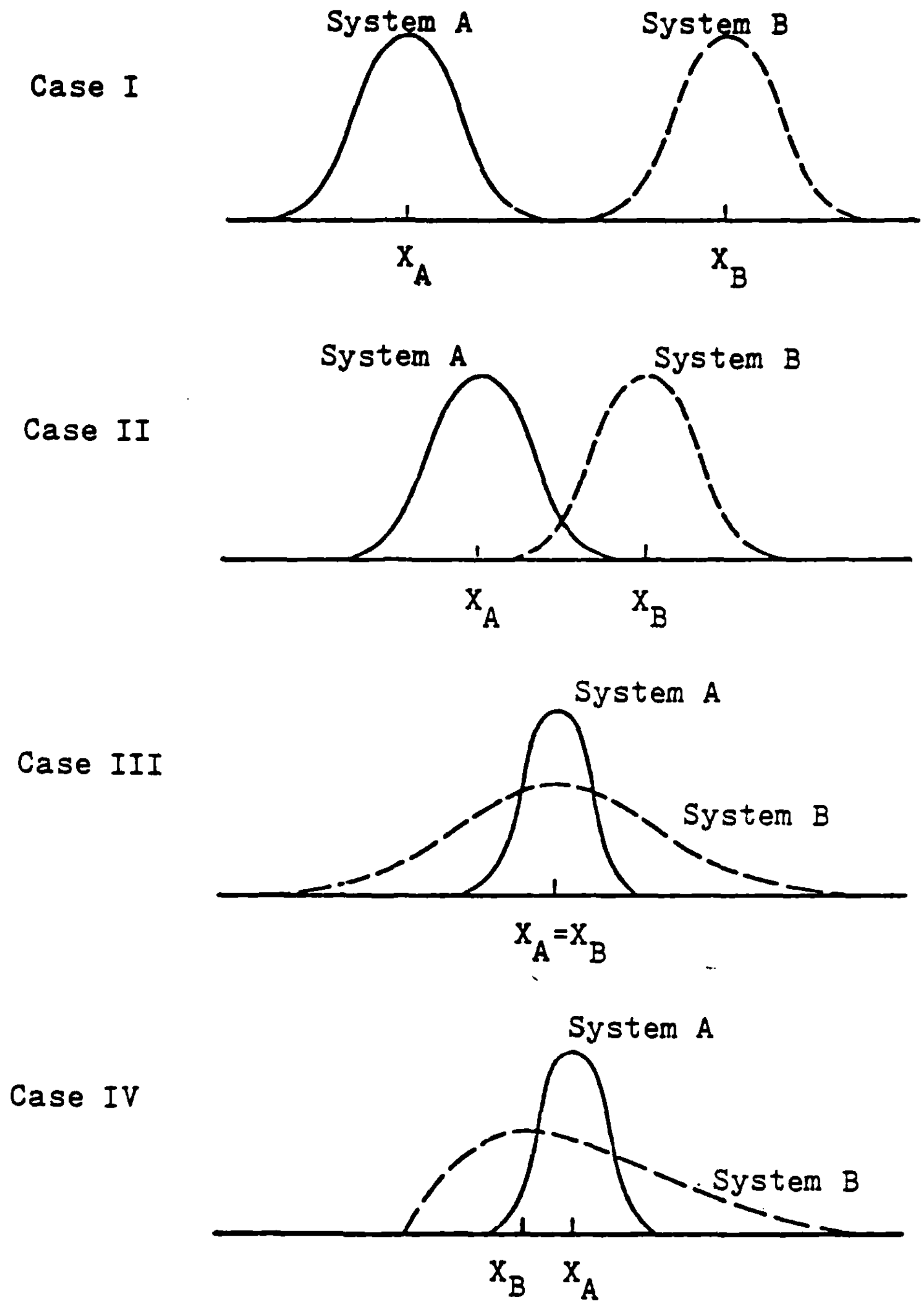


Figure 6: Impact of Cost Risk on Decision Making

were only interested in a point estimate. There were four predominate reasons for this. First, presenting more than a point estimate would constitute an information overload. Life cycle cost is but one input to the decision process. Information presented must be clear, concise, and easily understood. This leads to the second reason. Some interviewees believed that decision makers would not understand risk analysis and its associated implications. Third, some felt that the possibility of high costs would cause undue concern and adversely affect the decision. Fourth, risk analysis would impact the credibility of the study giving the impression that analysts were unwilling to stand behind their analyses. Most, however, did agree that analysts should do risk analysis for their own benefit and in support of the point estimate.

But, risk analysis provides precisely the information that the decision makers need. If alternatives cannot be clearly separated and evaluated on the basis of cost, if competing cost estimates fall within the error of the estimate, then the decision should be based on some criterion other than cost. If the probable cost range is too broad, steps should be taken to refine the estimate and decrease the range. Such steps include better data collection and improved estimating methods and techniques. If the possibility of high costs is so significant as to make the system

potentially unaffordable, decision makers should be aware of this prior to the decision. Ignoring such information does not lead to better decision making. On the contrary, it leads to cost overruns, unsupportable systems, and impaired readiness.

2.5 CONCLUSION

Table 2 summarizes the pertinent results of the interviews. The table shows the number of respondents, out of a total of eighteen, who identified a problem in a particular area. Some respondents voiced a problem in more than one area. The overall assessment in this chapter was based, however, on an integration of the personal interviews, literature search, and personal observations and experience of the author.

LCC is a valuable and viable tool for controlling escalating defense costs. If life cycle cost management is to reach its full potential, the credibility gap must be filled. This will require changes in business practice, organizational structure, and management philosophy. Also, steps must be taken to solve the problems associated with people, data, and modeling. In addition, risk analysis must be an integral part of every life cycle cost study and analysis. The next chapter discusses some of the modeling methods for meaningfully developing a risk analysis.

TABLE 2

Interview Summary

Problem	Number
Directives	1
Organization	6
Confusion with Budget	2
Program Stability	3
Program Manager	3
People	5
Data	9
Models	3
Risk Analysis Objection	9

Organization includes those who cited No Follow Up as a problem and Confusuion with Budget includes those who recognized Figure of Merit as a problem.

Chapter III

MODELING

To place a LCC estimate in proper perspective, it must be viewed as a random variable. Unfortunately, many users are not fully aware of this. But, it is this aspect of the LCC estimate which gives credence to risk analysis.

Although some of the cost factors or elements may be known with some degree of certainty and can be considered non-random, most are not known or identifiable with certainty and are, thus, random. In arriving at a LCC estimate, various quantities, random and non-random, are added, and multiplied, and, finally, aggregated. Any function of random variables is itself a random variable. Therefore, the final LCC is a random variable and must be viewed as such.

Each cost factor or element can be described by a probability distribution. The functional form of this distribution (e.g. normal, beta, gamma, etc.) may be known or unknown. In either case, certain parameters (e.g. mean, mode, median, variance, etc.) of the distribution may be known or unknown. Further, each cost element or factor may be depen-

dent or independent of the other cost factors or elements. Figure 7 illustrates this breakdown. Depending on the information available, various modeling methods are available to perform the risk analysis.

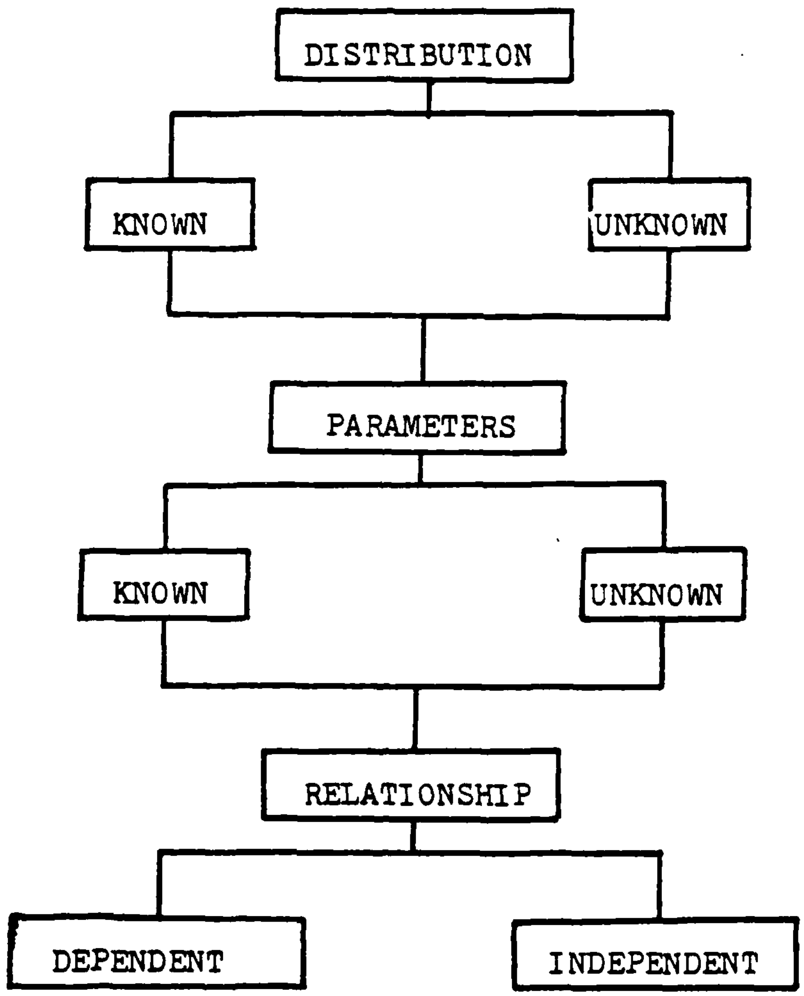


Figure 7: Cost Variable Breakdown

The major risk analysis effort should be directed toward the cost drivers, and, as established earlier, O&S cost is a driver with respect to life cycle cost. Therefore, the modeling presented here will be chiefly directed at O&S cost. However, the methodology and techniques to be developed and presented could be easily applied to acquisition costs.

This chapter begins with a theoretical review and discussion of several modeling methods and then relates two of these methods directly to O&S costing using the techniques of analogy and parametric costing. Some candidate probability distributions for use in O&S cost risk analysis are then presented. The chapter concludes with a discussion of various ways to present the risk analysis to decision makers.

3.1 MODELS AND MODELING METHODS

The models or mathematical expressions used in O&S costing appear in two general forms: the additive model and the multiplicative model. The additive model is expressed as

$$Y = C_1X_1+C_2X_2 \qquad (1)$$

and the multiplicative model as

$$Y = X_1X_2 \qquad (2)$$

where X_1 and X_2 are random variables and C_1 and C_2 are constants. The mathematical expressions used in practice appear to be more complex, but are usually reducible to these two general forms. These models are referenced frequently in the following discussion.

The modeling methods generally fall into two broad categories: analytical and Monte Carlo simulation. The method used in addressing a particular estimate depends upon the complexity of the problem itself and the amount and type of information available. It is conceivable that different parts of the analysis could be done with different methods. This section begins with a discussion of Monte Carlo simulation followed by a presentation of analytical methods.

3.1.1 Monte Carlo Simulation

Monte Carlo simulation is a method of estimating cost by means of an experiment with random numbers. Simulation involves replacing an actual statistical universe of cost elements and factors by its theoretical counterpart, a universe described by some assumed probability distribution, and then sampling from this theoretical population by means of some type of random number generator. This approach seeks answers to problems dealing with abstract, rather than real, populations and is ideally suited for situations where the taking of actual samples is either impossible or economically infeasible. Simulation is often used when problem complexity makes numerical analysis difficult, if not impossible, or when there are no known analytic solutions [90:241]. It is also used when explaining abstract analytical models to decision makers is too difficult.

Figure 8 illustrates the Monte Carlo approach. The cost factors, constants, and cost estimating relationship (CER) coefficients are treated as a set of inputs to the cost model. Associated with each of these inputs is a probability distribution to reflect its inherent risk. These distributions can be described statistically from available data or from subjective probabilities. The object is to estimate the total system cost and its associated uncertainty when all the input uncertainties are subjected to the complex interaction of the cost model. A simulation technique is used to generate the input parameters and a set of output (cost) estimates is prepared. From the set of output estimates, common statistical measures (mean, variance, range, etc.) and a frequency distribution are calculated.

As an illustration, consider the multiplicative model 2[12]. The analyst has determined (from actual data or subjectively) that the input parameter uncertainty is characterized by the probability distributions as shown in Figure 9 for X_1 and X_2.

From the input distributions, a sample value is generated using the Monte Carlo technique. By multiplying these sample values, a value for Y is calculated. The procedure is repeated until the nature of the output uncertainty has been

[12] An alternative model may be the additive model (1). The model actually used in a study should be that which best represents the reality of the situation. The model used here is for illustrative purposes only.

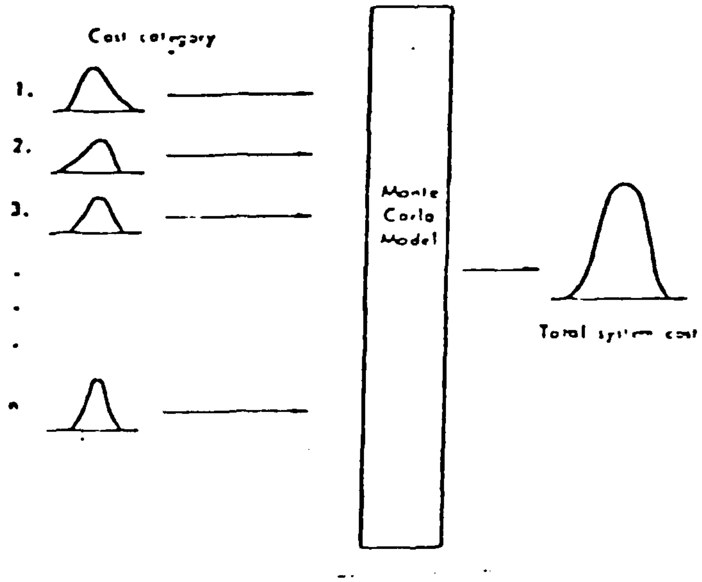


Figure 8: Monte Carlo Simulation Approach

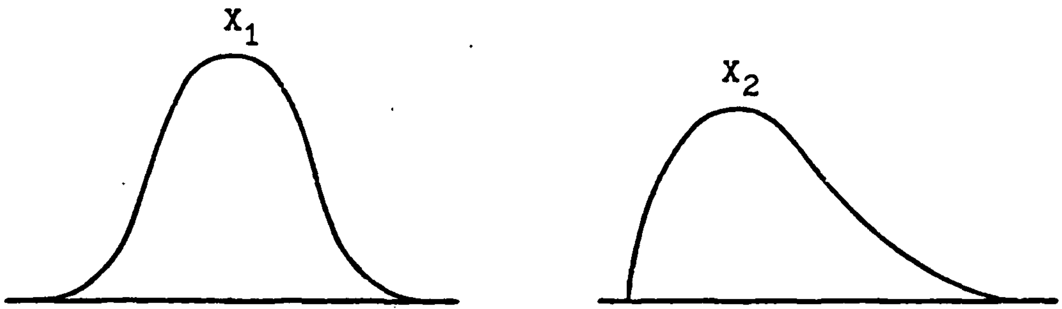


Figure 9: Simulation Example Distributions

established and a frequency distribution for Y can be drawn.

The Monte Carlo technique described herein requires that all inputs be mutually independent and, according to Dienemann [27:11], the cost factor inputs are probably independent. If they are not, Dienemann suggests incorporating the relationship into the cost model or using sophisticated techniques for sampling from joint frequency distributions. The major problem is in finding the proper form for the joint distributions or in defining the dependent relationship.

Monte Carlo simulation seems to be the most popular method of conducting a cost estimate risk analysis. Several references in the literature deal with this method[13]. The main advantage to using simulation is the ability to address complex problems. Although the example given here is quite simple, actual models may contain numerous equations and hundreds of variables. The disadvantages are dependence on a computer (no back of the envelope estimates) and inherent inexactness due to sampling error. However, with the type of models generally used in life cycle costing, a large sample size can easily be obtained. Thus, simulation can give fairly precise results. As an added bonus, simulation gives an estimate of the shape of the distribution. Additionally, simulation results are generally easily understood by man-

[13] The reader is referred to [23], [27], [40], [93], and [101].

agement.

3.1.2 Analytical Methods

Although not as popular in the literature as Monte Carlo simulation, some analytical methods have been applied with varying degrees of success to cost estimation problems. This author has found none, however, that specifically address the problem of O&S costing.

The analytical methods will generally yield exact solutions and are not necessarily computer dependent. However, the mathematics involved in the more sophisticated methods can be very complex with lengthy expressions to the point of making a solution almost impossible to obtain. In such cases, a computer may have to be used and approximations may be required.

Three methods are presented here. A short summary and review of additive and multiplicative moments is followed by an indepth presentation of sums and products of random variables and transforms. The remainder of this paper will emphasize the use of the last two analytical methods as they apply to O&S costing.

3.1.2.1 Additive and Multiplicative Moments

The concept of additive moments was first introduced by T. N. Thiele in 1903 under the name semi-invariants or cumu-

lants. The concept was later addressed by Cramer [25] and Kendall and Stuart [56]. The term additive moments was first used by S. A. Sobel in 1965. In 1976, McNichols [71] applied the concept to the treatment of risk in parametric costing. This procedure was later expanded by J. J. Black to include multiplicative moments. This section is based primarily on the work of McNichols and a paper by Wilder and Black [89:193-202].

By definition, the ith moment of the random variable X taken about the origin is $E[X^i]$ and is denoted by μ_i' where E stands for expected value. Mathematically,

$$\mu_i' = E[X^i] = \int_{-\infty}^{+\infty} X^i f(X)\ dX \tag{3}$$

where f(X) is the probability distribution function of the random variable X. The mean or expected value of the distribution of X, μ_1', is the first moment about the origin and will be denoted as μ. Expanding upon this definition, the ith moment of a random variable X taken about its mean, or the ith central moment of X, is $E[(X-\mu)^i]$ and is denoted by μ_i. Mathematically,

$$\mu_i = E[(X-\mu)^i] = \int_{-\infty}^{+\infty} (X-\mu)\quad f(X)\ dX \tag{4}$$

In particular, the variance of X, Var[X], is the second central moment, μ_2. The expected value of the random variable e^{itX} is called the characteristic function of the random

variable X. The kth derivative of the logarithm of the characteristic function evaluated at zero and multiplied by i^{-k} is the kth additive moment of the random variable. With these definitions in mind, Gnedenko [39:269] has shown that the first four additive or A moments of the random variable X, in terms of origin and central moments, are as listed in Table 3 . The reader will note that the first A moment is the same as the first moment about the origin. Also, the second and third A moments are the same as the second and third central moments. Thus, given a random variable X with known mean and central moments, the A moments can be found.

Further, the additive moments of a sum of independent random variables are equal to the sum of the additive moments of the individual terms [39:269]. Thus, referring to the additive model (1) with C_1 and C_2 equal to one, the A moments of Y are equal to the sum of the A moments of X_1 and X_2. Using the A moments of Y, it is then possible to solve for the parameters of a given distribution of Y [71:85-99].

For example, if one knows or assumes that the sum of two independent random variables is rectangularly distributed[14] then it can be shown that

$$A_1 = \mu = (2a+b)/2 \tag{5}$$

and

$$A_2 = \sigma^2 = b\,/12 \tag{6}$$

---

[14] See Table 10 .

where a and b are parameters of the distribution. Note that equations (5) and (6) are simultaneous equations in two unknowns. One can solve equation (6) for b and substitute that value into (5), yielding a corresponding value for a. That is,

$$b = +2\sqrt{3A_2} \tag{7}$$

$$a = A_1 - \sqrt{3A_2} \tag{8}$$

If one is dealing with sample data, sample mean and variance can be substituted in (5) and (6) above. For further examples, see McNichols [71: 85-99].

TABLE 3

Additive Moments

$$A_1 = E[X] = \mu$$

$$A_2 = E[X^2] - \mu^2 = \mu_2$$

$$A_3 = E[X^3] - 3E[X^2]\mu + 2\mu^3 = \mu_3$$

$$A_4 = E[X^4] - 4E[X^3]\mu - 3(E[X^2])^2 + 12E[X^2]\mu^2 - 6\mu^4$$

For the multiplicatve model (2) where X_1 and X_2 are independent random variables, Sobel [89:195] has shown that the moments about the origin of the individual random variables can be multiplied to get the moments about the origin of the

product. These multiplicative or M moments, where M_i is the ith multiplicative moment, can then be converted to A moments and vice versa as given in Table 4 and Table 5 [89:195].

TABLE 4

Additive to Multiplicative Moments

$$M_1 = A_1$$
$$M_2 = A_2+A_1^2$$
$$M_3 = A_3+3A_1A_2+A_1^3$$
$$M_4 = A_4+3A_2^2+4A_1A_2+6A_1^2A_2+A_1^4$$

TABLE 5

Multiplicatve to Additive Moments

$$A_1 = M_1$$
$$A_2 = M_2-M_1^2$$
$$A_3 = M_3-3M_1M_2+2M_1^3$$
$$A_4 = M_4-3M_2^2-4M_1M_2+12M_1^2M_2-6M_1^4$$

The procedure can be easily and obviously extended for the sum and product of more than two random variables. It can also be applied to mixed additive and multiplicative models[15] through the application of the relationships in Tables 4 and 5.

For models more complex than the simple additive and multiplicative models with unit coefficients given in (1) and (2) and the extensions noted above, more complex procedures must be used. Wilder and Black use a transformation of variables to simplify more complex forms [89:195-196]. McNichols uses a Taylor series expansion of the model and computes the additive moments for first and second order approximations [71:77-83]. Thus, many functional forms of the model can be addressed using additive and multiplicative moments.

The discussion of additive and multiplicative moments has thus far concerned only independent random variables. Worm [105] extends the McNichols methodology to include the dependent case by splitting each random variable into two parts: one representing the independent part and the other the dependent part. Considering the additive model (1), one must estimate for each X_i a proportion of the total variation due to a commonality between the X_i's. This information is used to split the variance of the X_i into the two

[15] $Y = X_1(X_2+X_3)$ and $Y = (X_1X_2)+(X_3X_4)$ are examples of mixed additive and multiplicative models.

parts. Each X_i is then considered to be the sum of two independent random variables, one of the two being a part of the other X_i. Using these two random variables in place of the X_i makes Y a function of independent random variables. The additive moments are computed in accordance with the procedure presented by McNichols. McNichols also presents a procedure for determining the additive moments for the additive model (1) under conditions of dependency [71:116-117]. In applying his procedure, the analyst must determine the covariance between each combination of dependent variables.

Although both procedures for the dependent case may sound simple and direct, it must be remembered that the proportion of commonality or covariance must be determined either from historical data or subjective input. This may be no trivial task if sufficient data is not available or if reliable subjective input cannot be collected.

3.1.2.2 Sums and Products of Random Variables

The sums and products of random variables method is the most elementary of the analytical methods. Perhaps this explains why this method seems to be overlooked in the literature. It requires the least amount of information relative to the random variables involved, but does necessitate some basic assumptions in the application of the statistical techniques. The sums and products of random variables is applica-

ble in both the independent and dependent cases and requires only a knowledge of the mean, variance, and, where applicable, covariance of the quantities involved.

Appropriate formulae for the independent and dependent case of the additive model (1) are contained in Table 6 ; while corresponding formulae for the multiplicative model (2) are contained in Table 7 .

TABLE 6

Additive Model Formulae

$$Y = C_1X_1 + C_2X_2$$

Independent Case

Mean

$$\mu_Y = C_1\mu_1 + C_2\mu_2$$

Variance

$$\sigma_Y^2 = C_1^2\sigma_1^2 + C_2^2\sigma_2^2$$

Dependent Case

Mean

$$\mu_Y = C_1\mu_1 + C_2\mu_2$$

Variance

$$\sigma_Y^2 = C_1^2\sigma_1^2 + C_2^2\sigma_2^2 + 2C_1C_2\sigma_{12}$$

Note: μ_i = Expected value of X_i = $E[X_i]$

σ_i^2 = Variance of X_i = $Var[X_i]$

σ_{ij} = Covariance of X_i and X_j = $Cov[X_i, X_j]$

TABLE 7

Multiplcative Model Formulae

$$Y = X_1 X_2$$

Independent Case

Mean

$$\mu_Y = \mu_1 \mu_2$$

Variance

$$\sigma_Y^2 = \mu_1^2 \sigma_2^2 + \mu_2^2 \sigma_1^2 + \sigma_1^2 \sigma_2^2$$

Dependent Case

Mean

$$\mu_Y = \mu_1 \mu_2 + \sigma_{12}$$

Variance

$$\sigma_Y^2 = \mu_1^2 \sigma_2^2 + \mu_2^2 \sigma_1^2 + 2\mu_1 \mu_2 \sigma_{12} + \sigma_1^2 \sigma_2^2$$

Note: μ_i = Expected value of X_i = $E[X_i]$

σ_i^2 = Variance of X_i = $Var[X_i]$

σ_{ij}= Covariance of X_i and X_j = $Cov[X_i, X_j]$

The additive model can be easily extended to accomodate the general case of the sum of n random variables. Formulae for the general case are contained in Table 8 .

Extension of the multiplicative model to the general case is not so easily accomplished and, therefore, as a practical matter, an approximation using a Taylor series expansion of the model is recommended. This procedure also applies to

TABLE 8

General Additive Model Formulae

$$Y = C_1X_1+C_2X_2+\ldots+C_nX_n$$

Independent Case

Mean

$$\mu_Y = \sum_{i=1}^{n} C_i\mu_i$$

Variance

$$\sigma_Y^2 = \sum_{i=1}^{n} C_i^2\sigma_i^2$$

Dependent Case

Mean

$$\mu_Y = \sum_{i=1}^{n} C_i\mu_i$$

Variance

$$\sigma_Y^2 = \sum_{i=1}^{n} C_i^2\sigma_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} C_iC_j\sigma_{ij}$$

Note: μ_i = Expected value of X_i = $E[X_i]$

σ_i^2 = Variance of X_i = $Var[X_i]$

σ_{ij} = Covariance of X_i and X_j = $Cov[X_i, X_j]$

$\sum$ = Summation operator

compound functions of random variables, particularly in the dependent case. This method is sometimes referred to in the literature as the method of statistical differentials.

Consider a function of n variables, $f(X_1, X_2, ... X_n)$, where each of the variables has a known mean, variance, and, where applicable, covariance. Taylor's formula for the function of n variables about the point $P = (a_1, a_2, ... a_n)$ is

$$f(X_1,X_2,\ldots,X_n) \doteq f(a_1,a_2,\ldots,a_n) + \sum_{i=1}^{n}(X_i-a_i)\ \partial f/\partial X_i|_P$$
$$+ 1/2\sum_{i=1}^{n}\sum_{j=1}^{n}(X_i-a_i)(X_j-a_j)\ \partial^2 f/\partial X_i\partial X_j|_P$$
$$+ R \qquad (9)$$

where $\partial f/\partial X_i$ and $\partial^2 f/\partial X_i \partial X_j$ are the first and second partial derivatives evaluated at $P = (a_1, a_2, \ldots, a_n)$ respectively and R is a remainder term. Assuming R is negligible,[16] and substituting $P_\mu = (\mu_1, \mu_2, \ldots, \mu_n)$ for P where μ_i is the expected value of X_i, it can be shown that the approximate expected value of the function f is

$$E[f(X_1,X_2,\ldots,X_n)] \doteq f(\mu_1,\mu_2,\ldots,\mu_n)$$
$$+1/2\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}\ \partial^2 f/\partial X_i\partial X_j|_{P_\mu} \qquad (10)$$

where σ_{ij} is the covariance for X_i and X_j. If X_i and X_j are independent, the last term is zero for the X_i, X_j pair. The variance of $f(X_1, X_2, \ldots, X_n)$ is approximated by

$$Var[f(X_1,X_2,\ldots,X_n)] \doteq \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}\ (\partial f/\partial X_i|_{P_\mu})(\partial f/\partial X_j|_{P_\mu})$$
$$(11)$$

---

[16] R can be safely disregarded if third order derivatives are small or zero.

The major difficulty in applying the sums and products of random variables is in determining values for the mean, variance, and, where applicable, covariance of the random quantities concerned. This problem will be addressed in later sections of this chapter.

3.1.2.3 Transforms

This method does offer some features and versatility not found in the other analytical methods presented. Transform techniques can be used to generate the origin and central moments necessary in the application of additive and multiplicative moments or as a stand alone analytical technique. In the same way that logarithms are used to manipulate complicated expressions in elementary algebra, transform are used to simplify operations in the algebra of random variables. The use of transforms requires no more assumptions and little more knolwedge than the method of additive and multiplicative moments, yet affords an added bonus in that the resultant expression can be used to reveal the exact distribution of the product or sum of random variables. If one is only concerned with the moments of that distribution, they are obtainable directly from the transform without going through the sometimes difficult task of transform inversion.

Three transform types will be discussed in this section. These are the Laplace transform for the additive model (1), the Mellin transform for the multiplicative model (2), and the geometric, in conjunction with the Laplace, for a special case of the additive model.

The transform most used in finding the distribution of sums of continuously distributed non-negative random variables is the Laplace transform. Given a continuous random variable X, $(X \geq 0)$, with probability density function f(X),

$$L[f(X)] = \int_0^{\infty} e^{-sX} f(X)\ dX \qquad (12)$$

is the unilateral Laplace transform of f(X), where s is a complex variable of the form $s = a + iw$. Its inversion integral is

$$L^{-1}[f(X)] = f(X) = (2\pi i)^{-1} \lim_{b \to \infty} \int_{a-ib}^{a+ib} e^{sX} L[f(X)]\ ds \qquad (13)$$

Equations (12) and (13) above constitute the Laplace transform pair. Note that the inversion requires the evaluation of a contour integral. This evaluation can be avoided by the use of tables of transform pairs such as those found in Giffin [38] and Springer [97].

In multiplying random variables, the Mellin transform is used to develop probability distributions. The Mellin transform for the probability density function f(X) of the random variable X is

$$M[f(X)] = \int_0^{\infty} X^{s-1} f(X)\ dX \qquad (14)$$

where s is once again a complex variable. The Mellin inversion integral is

$$M^{-1}[f(X)] = f(X) = (2\pi i)^{-1} \lim_{b\to\infty} \int_{a-ib}^{a+ib} X^{-s} M[f(X)]\, ds \qquad (15)$$

Equations (14) and (15) above are the Mellin transform pair. Again, the inversion requires the evaluation of a contour integral. As in the case of the Laplace transform, this integration can be avoided by using tables such as those in Giffin [38] and Springer [97].

In order to properly evaluate the additive and multiplicative models, one must apply the convolution operation. Consider the additive model (1), where X_1 and X_2 are continuous independent random variables with probability density functions $f_1(x_1)$ and $f_2(x_2)$ respectively. The the distribution of Y is given by convolution integral

$$g(Y) = \int_0^Y f_1(X)\, f_2(Y-X)\, dX = f_1 * f_2 \qquad (16)$$

where the symbol * represents the convolution operation. In the transform domain, the convolution of two functions, according to Borel's theorem, is equal to the inverse of the product of their transforms. Simply stated in terms of the Laplace transform,

$$L[g(Y)] = L[f_1(X_1)]\, L[f_2(X_2)] \qquad (17)$$

or the transform of the probability density function of a sum is the product of the transforms of the density functions of the individual random variables being added. This procedure can be logically extended to the sum of n continuous independent random variables

$$Y = X_1 + X_2 + \ldots + X_n \tag{18}$$

as

$$L[g(Y)] = \prod_{i=1}^{n} L[f_i(X_i)] \tag{19}$$

where π is the multiplication operator. If the n terms in the sum are all identically distributed independent random variables, (19) reduces to

$$L[g(Y)] = (L[f(X)])^n \tag{20}$$

If the inversion of the transform of g(y) proves to be too difficult or if one is only interested in the moments of the distribution of g(Y), all is not lost. In the case of the Laplace transform, the moments of the distribution can be determined by simply taking derivatives of the transform with respect to s and evaluating at s=0 [38:62-63]. In mathematical terms,

$$d^rL[f(X)]/ds^r|_{s=0} = \int_0^\infty (-X)^r e^{-sX} f(X)\, dX \,|_{s=0} \tag{21}$$

and the moments about the origin can be expressed as

$$E[X^r] = (-1)^r\, d^rL[f(X)]/ds^r|_{s=0} \tag{22}$$

If one is interested in the moments about the mean, , the logic above can be extended. That is,

$$E[(X-\mu)^n] = (-1)^n \, d^n/ds^n (e^{s\mu} L[f(X)]) |_{s=0} \qquad (23)$$

Thus, the variance and higher order moments about the mean can be easily and quickly determined given the distribution function of the random variable x.

Now consider a special case of the Laplace transform where the number of terms in the sum is itself a random variable. This situation is conveniently handled through the application of the geometric transform.

The geometric transform is the discrete analog of the Laplace transform and provides the same capabilities with respect to discrete random variables as the Laplace with continuous random variables. The geometric transform is defined by the sum

$$G[p(n)] = \sum_{n=0}^{\infty} p(n) z^n = G_n(z) \qquad (24)$$

where p(n) is the probability mass function of n and z is a complex transform variable corresponding to s in the Laplace transform. Thus, the geometric transform of the function p(n) can be viewed as the expected value of z^n. The expression in (18) can then be treated as a conditional transform. That is, $L[g(Y|n)]$ is the transform of g(Y) for some fixed n. The unconditional transform is then obtained by weighting each term by p(n) and summing over all n. That is,

$$L[g(Y)] = \sum_{n=0}^{\infty} L[g(Y|n)] p(n) = \sum_{n=0}^{\infty} (L[f(X)])^n p(n) \qquad (25)$$

Note that this expression closely resembles that of the geometric transform (24) with the z replaced by $L[f(x)]$. The transform for the random sum of random variables is then expressed by

$$L[g(Y)] = G_n(z)\big|_{z=L[f(X)]} = G_n[L[f(X)]] \tag{26}$$

Although this expression appears to be a geometric transform, it is not. The end result is a Laplace transform in the complex variable s and must be treated as such.

Often in O&S costing problems, historical data or subjective estimates are so sparse or incomplete as to permit only an estimate of the mean and variance of the random variables x and n. In these cases where there is insufficient, credible information to fit distributions to the random variables involved, one can still solve for the mean and variance of the random sums expressed in equation (18). That is,

$$\mu_Y = \mu_n \mu_X \tag{27}$$

and

$$\sigma_Y^2 = \mu_n \sigma_X^2 + \mu_X^2 \sigma_n^2 \tag{28}$$

Thus, given only the mean and variance of both the number of terms and the individual terms in the sum, one can compute the mean and variance of the random sum. One need not know the distribution or the transform to obtain these important statistics. Equations (27) and (28) hold for both continuous and discrete X.

The Mellin convolution used in the multiplicative model takes a slightly different form. The probability density function of the product of two nonnegative, continuously distributed, independent random variables, model (2), with probability density functions $f_1(x_1)$ and $f_2(x_2)$ is, in terms of the Mellin convolution,

$$g(Y) = \int_0^{\infty} 1/X \; f_1(Y/X) \; f_2(X) \; dX \qquad (29)$$

Then, the transform of the Mellin convolution is

$$M[g(Y)] = M[f_1(X_1)] \, M[f_2(X_2)] \qquad (30)$$

where M represents the Mellin transform of the densities involved. In parallel with the sum of n continuous independent random variables given above, the transform of the density function of the product of n nonnegative independent random variables is

$$M[g(Y)] = \prod_{i=1}^{n} M[f_i(X_i)] \qquad (31$$

The recovery of the moments of the Mellin transform is even easier than the Laplace and does not involve derivatives but a mere substitution of variables. It can be shown that the rth moment about the origin can be derived by replacing the complex variable s in the Mellin transform with s = r+1 [38:74]. In mathematical symbols,

$$E[X^r] = M[f(X)]|_{s=r+1} = \mu_r' \qquad (32)$$

If one is interested in central moments rather than moments about the origin as expressed in (32), the conversions given in Table 9 can be used.

TABLE 9

Conversion - Origin to Central Moments

$$\mu_1 = \mu_1'$$
$$\mu_2 = \mu_2' - (\mu_1')^2$$
$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$$
$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4$$

This discussion of transforms has thus far dealt with only the independent case. Information relative to the dependent case is rather lacking. This is primarily due to the fact that the analysis is considerably more complicated. For the additive model (1), the transform of the probability density function of the sum of dependent random variables is no longer the product of the transforms of the probability density functions of the component random variables [97:67]. Springer [97] uses the Fourier transform or multivariate characteristic function to evaluate the sum of dependent random variables. To apply the technique, one must know the multivariate probability density functions. He then demon-

strates the technique for the sum of dependent normal random variables.

For the multiplicative model (2), Springer [97] uses the two dimensional Mellin transform to evaluate the product of two dependent random variables. Again, one must know the multivariate probability density function. Springer demonstrates this procedure using the bivariate normal distribution.

The fundamental problem in the dependent case is that evaluation of both the additive and multiplicative models requires a knowledge of the multivariate probability density function. Given the present state of the art in O&S costing, it seems unlikely that these distributions can be identified with any degree of confidence. Therefore, one may have to resort to the methods discussed in the section on sums and products of random variables. This is a natural for a Taylor series approximation. The consequence of ignoring dependence is an error in the variance for the additive model and an error in both the expected value and variance for the multiplicative model. The magnitude of the error is a function of the magnitude of the covariance and, where applicable, the expected value of the dependent variables involved.

3.2 MODELING O&S COSTS

In this section, the theoretical methods and models presented in the previous sections are related directly to the problem of O&S costing. The techniques of analogy and parametric costing are emphasized as they are the techniques used in the early stages of a weapons system acquisition program. The reader is reminded that it is the early decisions which have the greatest impact on total cost.

3.2.1 The Basic Building Block

The basic building block for any total system O&S cost estimate is the single year O&S cost. These yearly estimates afford a good frame of reference and conform to the widely used practice of collecting, processing, and aggregating data on a yearly basis.

The key to determining single year O&S cost is the cost element structure (CES). As the reader will recall, the CES is simply a listing of the cost items or categories to be included in the yearly estimate. In using an accounting type model, the cost elements are then added, often in a hierarchical fashion, to arrive at the cost for year j, Y_j. Thus, if X_i is the cost estimate for the ith cost element in support year j,

$$Y_j = \sum_{i=1}^{m} X_i \, , \quad j=1,\ldots,n \tag{33}$$

where m is the number of cost elements and n is the operational life of the system.

Then, if the X_i's are independent, using simple sums of random variables, the expected value and variance of Y_j are

$$E[Y_j] = \sum_{i=1}^{m} E[X_i] \tag{34}$$

and

$$Var[Y_j] = \sum_{i=1}^{m} Var[X_i] \tag{35}$$

If the X_i's are dependent, the variance is expressed as

$$Var[Y_j] = \sum_{i=1}^{m} Var[X_i] + 2\sum_{i=1}^{m-1} \sum_{j=i+1}^{m} Cov[X_i, X_j] \tag{36}$$

where Cov is the covariance.

In terms of transforms, the Laplace transform of the distribution of Y_j, $g(y)$, is

$$L[g(Y_j)] = \prod_{i=1}^{m} L[f_i(X_i)] \tag{37}$$

where $L[f_i(x_i)]$ is the Laplace transform of the distribution of X_i. Note that the above transform applies only in the independent case.

Now that a method for estimating yearly cost has been established, how is total O&S cost over the life cycle of the system computed?

3.2.2 Total O&S Cost

The single year cost may represent a steady state condition under which the number of systems in operation is rather fixed and costs remain relatively steady or it may be for a year characterized by the phase in or phase out of the system. The typical O&S cost profile is shown in Figure 10 .

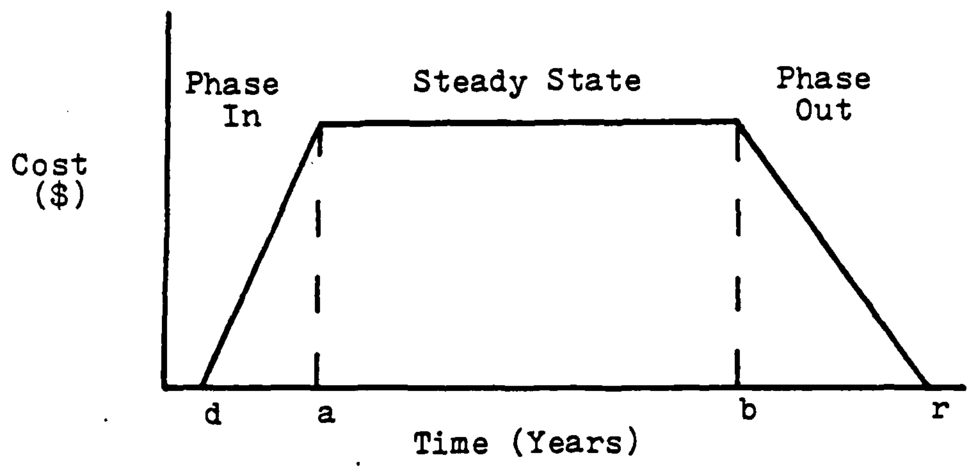


Figure 10: Typical System O&S Cost Profile

A weapons system begins accruing O&S cost when the first item or member of the system is introduced into the active inventory. The initial period, d to a in Figure 10 , is known as phase in and continues until the last production model enters the active inventory. The cost curve during this time has a positive slope indicating an increasing cost

trend with the actual shape and slope of the curve being a function of the production schedule in terms of rate and quantity.

Following phase in, the system enters a period of relatively stable costs known as steady state, shown as a to b in Figure 10 . At the end of steady state, phase out begins and continues until the last active item is retired, as shown from b to r in Figure 10 . This is a period of decreasing O&S costs and is again a function of schedule in terms of rate and quantity. Note that phase in and phase out may occur at different rates. The area under the curve is then the total O&S cost.

A common and rudimentary approach used in determining total O&S cost is to compute the single year steady state cost for the system and then multiply that cost by a fixed number of years. This approach has several faults and is really valid only if the phase in and phase out rates are approximately the same. If the steady state cost is used to represent all the years included in the estimate, a better model may be to add the cost of the individual years. Another approach is to compute separate costs for the phase in years and then add on the cost for a fixed number of steady state years ignoring the phase out years altogether. As is obvious, this approach also has faults from the standpoint of capturing the total O&S cost. However, paramount in any

cost analysis is the decision for which the estimate is being made. The analysis must be relevant to the decision and convey to decision makers that information which will enlighten and support the decision. Therefore, these simplified techniques may be all that are required for the decision at hand. To be exact, however, total lifetime O&S cost for a system can be represented by the equation

$$TOS = \sum_{j=d}^{r} Y_j \tag{38}$$

where TOS is the total O&S cost, Y_j is the O&S cost for support year j, d is the year in which support begins, and r is the year in which support ends. The values of d and r are based on the time reference for the study with d not necessarily equal to one. In that TOS is a function of time, it becomes necessary to discuss two issues before going further. These issues are inflation and discounting; both of which modify the relationship of future to present costs.

3.2.2.1 Inflation

In recent years, monetary inflation has been a worldwide phenomenon and it is expected to continue. In simple terms, monetary inflation describes a period of rising prices for goods and services. As prices rise, the value of the dollar falls. When developing cost profiles over time, the aspect of inflation must be considered.

Air Force Regulation 178-1 states that to detect the effect of changes in the purchasing power of the dollar, both constant dollars (adjusted for inflation to a base year, usually the present year) and current dollars (actual, future dollars) be considered in making cost estimates and in analyzing and evaluating alternatives. Further guidance from the Cost Analysis Improvement Group, Office of the Secretary of Defense, states that O&S costs should be presented in constant dollars of the current fiscal year. The Group reasons that constant dollars make future costs look more reasonable and give decision makers a benchmark for comparison. Many decision makers are also involved in the budgeting process and constant dollars give a frame of reference in relation to the other cost figures with which they are used to working. From the standpoint of both decision makers and analysts, inflation is difficult to predict and beyond their control. The use of constant year dollars eliminates the consideration of such uncertainty from the decision process. Thus, the use of constant dollars is highly recommended in O&S cost analysis. Various indices are available for converting one year's dollars to another. The Department of Defense recommended tables for O&S costs are found in Air Force Regulation 173-13, USAF Cost and Planning Factors. After accounting for the effect of inflation, another difference between future and constant dollars still remains.

3.2.2.2 Discounting

Discounting is a technique which accounts for the time value of money. A dollar in hand today is worth more than a dollar to be received at some time in the future because money has a cost - interest.

By discounting, all time phased expenditures are indexed to the present, the only fair method of evaluation if the decision is being made today. In other words, discounting relates all costs to a specific decision point. The effect of discounting on O&S costing is to make alternatives which defer spending more attractive than those which require spending sooner. Thus, if two alternatives are equal in all respects except in the timing of the same total expenditures, that alternative which delays spending is preferable. Discounting is required when comparing alternatives and in evaluating two or more cost streams on a comparable basis.

There are those, however, who argue against discounting. The LCC method encourages program managers to spend additional money on research and development and during production so as to save money later on O&S costs during the deployment phase. To them, discounting seems to diminish this rationale. The justification for discounting comes when one looks beyond the Department of Defense and at the federal government as a whole. Better, safer, surer investment opportunities should be the criterion by which the time value

of money is judged. This is called opportunity cost. The return on government investments is difficult to determine because decisions are not profit motivated. Often, government spending leads to intangible benefits. However, benefits are expected to exceed costs and, thus, discounting is applicable in the government sector. One need look no further than the enormous, mounting federal deficit to recognize an alternative use of money. The government always has the option to repay its debt, thus saving future interest costs. If discounting is not used, it implies that there are no investment opportunities offering a gain or that the interest rate is zero. In relation to the deficit, neither of these is true. Thus discounting is recommended. If increased spending up front to save support costs later is justified with discounting, then one can be assured that this is the proper decision. Otherwise, those additional funds could be used to retire the debt and assure some savings at the prevailing interest rate.

Once the application of discounting has been justified, a secondary issue is what rate to use. Present Department of Defense directives recommend a rate of ten percent. According to Air Force Regulation 178-1,

> The discount rate reflects the preference for current and future money sacrifices that the public exhibits in non-Government transactions. Since a 10 percent rate is considered to be the most representative overall rate at the present time, future costs will be discounted at a annual rate of 10 percent. The prescribed discount rate of 10

> percent represents an estimate of the average rate of return on private investment before corporate taxes and after adjusting for inflation.

A more appropriate rate may be the interest rate the government pays on borrowed money. Whatever the rate, once the mathematics of discounting are established, any rate can be quickly and easily substituted. Decision makers may even want to see several rates used.

Now that the application of discounting has been explained, using the sums and products of random variables presented earlier, the discounted sum or present value of TOS is expressed as

$$PV(TOS) = \sum_{j=d}^{r} (1+I)^{-j} Y_j \tag{39}$$

where I is the discount rate. Then the expected value and variance of the present value are, respectively,

$$E[PV(TOS)] = \sum_{j=d}^{r} (1+I)^{-j} E[Y_j] \tag{40}$$

$$Var[PV(TOS)] = \sum_{j=d}^{r} (1+I)^{-2j} Var[Y_j] \tag{41}$$

if the Y_j's are independent and

$$E[PV(TOS)] = \sum_{j=d}^{r} (1+I)^{-j} E[Y_j] \tag{42}$$

$$Var[PV(TOS)] = \sum_{j=d}^{r} (1+I)^{-2j} Var[Y_j]$$

$$+2\sum_{j=d}^{r-1}\sum_{k=j+1}^{r} \mathrm{Cov}[Y_j, Y_k](1+I)^{-(j+k)} \quad (43)$$

if the Y_j's are dependent where Cov stands for covariance.

In the transform domain, if the Y_j's are independent, the transform of the distribution of TOS is

$$L[g(TOS)] = [L[f_1(Y)] \cdots L[f_n(Y)]]|_{s_j=(1+I)^{-j}s} \quad (44)$$

where the complex variable s is replaced by $[(1+I)^{-j} s]$ for the jth transform on the right hand side of (44).

At his time, one problem remains. How does one compute the individual cost elements? This is the point at which analogy and parametrics enter the O&S costing picture.

3.2.3 Individual Cost Elements

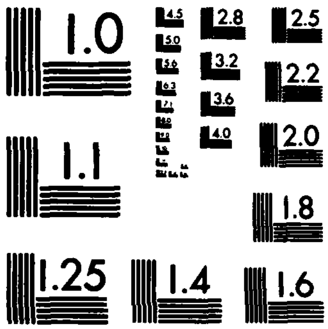
Two techniques for arriving at values for the individual cost elements are analogy and parametric costing. In using analogy, the reader will remember that an existing baseline system is first identified. The new system is then compared to the old, similarities and differences are noted, and adjustments made to the historical costs of the existing system to reflect these similarities and, even more so, differences. Thus, the cost of element X can be expressed as

$$X = \text{Base} \pm \text{Change} = W \pm Z \quad (45)$$

where Base (W) is the yearly cost for element X of the baseline system and Change (Z) is the adjustment made to reflect the new system. Over time, cost element X for the baseline system will fluctuate from year to year. Depending on the

AD-A133 023 LIFE CYCLE COSTING IN A DYNAMIC ENVIRONMENT(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH J A LONG 1983 AFIT/CI/NR-83-26D 2/3

UNCLASSIFIED F/G 5/1 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

modeling situation and the perspective of the analyst, this fluctuation can be treated in one of two ways. Consider the time series f(t) of yearly costs for the element X shown in Figure 11. One can take the collection of sample points, move or collapse them to the origin, fit an appropriate probabilty distribution to the resulting frequency distribution, and/or compute an expected value and variance without regard to the time element. (See Figure 12.) In doing so, one is assuming that that f(t) is the product of a stationary, ergodic stochastic process. In this case, W is a random variable.

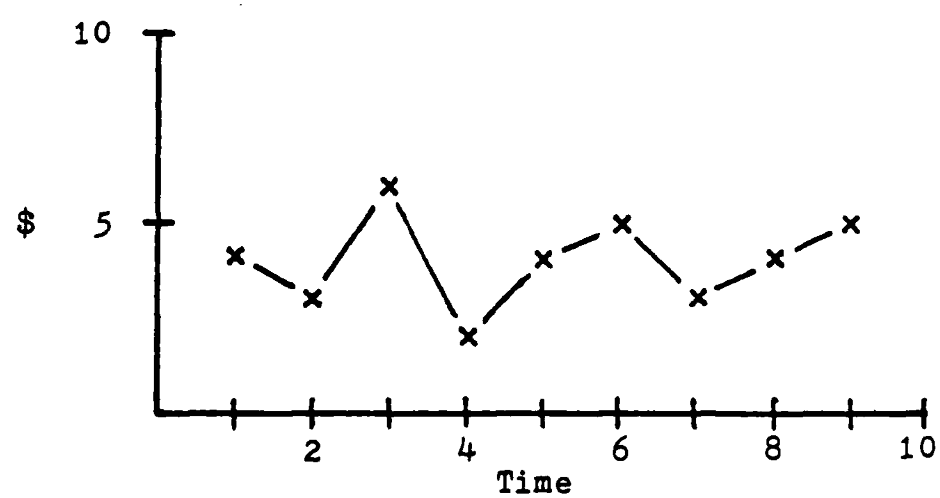


Figure 11: Time Series Example

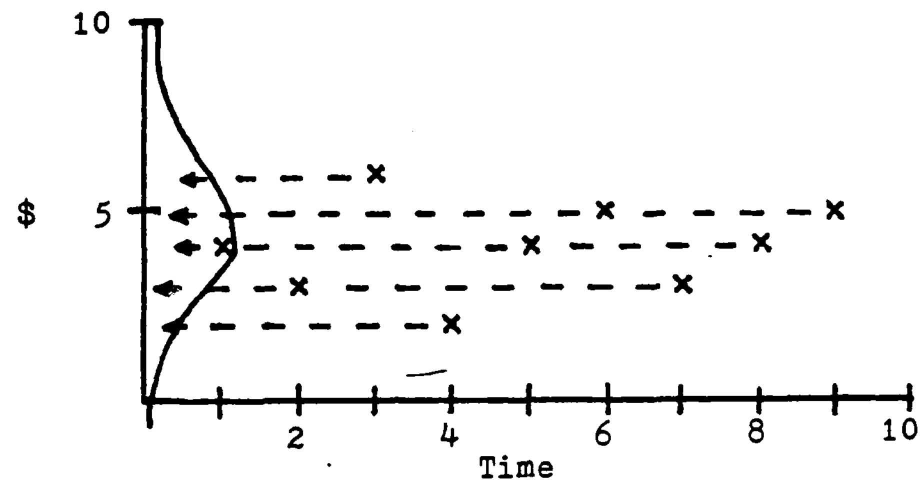


Figure 12: Time Series Collapsed

By the same token, the value of Change (Z) is not known with certainty. Thus it also can be regarded as a random variable from some possibly unknown distribution. Using the sums and products of random variables approach, and assuming W and Z are independent, the expected value and variance of the cost element X are

$$E[X] = E[W] \pm E[Z] \tag{46}$$

$$Var[X] = Var[W] + Var[Z] \tag{47}$$

Note that the variance is a sum for both the sum and difference of two random variables. In terms of transforms, the distribution of X can be expressed as a convolution integral and applying Borel's theorem,

$$L[f(X)] = L[g(W)]\,L[h(Z)] \tag{48}$$

where $L[g(W)]$ and $L[h(Z)]$ are the Laplace transforms of the distributions of W and Z respectively. In the case of a difference, the complex variable s in $L[h(Z)]$ is replaced by minus s $(-s)$.

As the second alternative in treating the time series, one can look at the data points as a single realization of the time series (and the only realization available) and compute a separate cost for the element X for each year. This treatment is applicable if the time series appears to be changing over time; that is, a positive or negative trend is apparent. In the absence of any further information, the value of W for any year j is regarded as a constant. Thus,

$$X_j = W_j \pm Z_j \tag{49}$$

The change in the cost element in year j, Z_j, is still a random quantity. Thus, the expected value and variance of X_j are, respectively

$$E[X_j] = W_j \pm E[Z_j] \tag{50}$$

$$Var[X_j] = Var[Z_j] \tag{51}$$

In transform domain, the Laplace transform of the distribution of X_j is

$$L[f(X_j)] = e^{-sW_j}\,L[h(Z_j)] \tag{52}$$

Again, in the case of a difference, the complex variable s in $L[h(Z)]$ is replaced by minus s $(-s)$.

Parametric estimation presents a more complex situation due to the complexity of some of the CERs used. CERs considered here are of two types: factor based and regression analysis based. The factor based CERs are essentially simple sums and products of random variables. As an example, consider a CER where fuel cost per year is estimated by multiplying flying hours per year by fuel cost per flying hour or

$$FC = FH \times CFH \tag{53}$$

where

FC = fuel cost per year

FH = flying hours per year

CFH = fuel cost per flying hour

Regarding FH and CFH as random variables, FC is then the product of two random variables. FH and CFH are factors. Thus, in general

$$X_i = \prod_{k=1}^{q} F_k, \quad i=1,\ldots,m \tag{54}$$

where q is the number of factors in the product for the ith cost element. Cost factors may also be added[17]. Thus, a CER may appear as

$$X_i = \sum_{k=1}^{q} F_k, \quad i=1,\ldots,m \tag{55}$$

---

[17] In cost estimation, factors are not defined in the algebraic sense of quantities multiplied together.

where q is the number of terms in the sum. Multiplicative and additive factors may appear in the same CER. To properly evaluate these CERs, the reader is referred to the sections on sums and products of random variables and transforms.

Before proceeding with regression analysis based CERs, a word of caution is in order. Consider the two alternative CERs

$$\text{Example 1} \qquad Y = 4X \tag{56}$$

and

$$\text{Example 2} \qquad Y' = X+X+X+X \tag{57}$$

Although they may appear to be the same at first glance, they are not. The expected values of Examples 1 and 2 respectively are

$$E[Y] = 4E[X] \tag{58}$$

and

$$E[Y'] = E[X]+E[X]+E[X]+E[X] = 4E[X] \tag{59}$$

which reinforces the notion that the models are the same, but the variences of Examples 1 and 2 respectively are

$$Var[Y] = 4^2 Var[X] = 16Var[X] \tag{60}$$

and

$$Var[Y'] = Var[X]+Var[X]+Var[X]+Var[X] = 4Var[X] \tag{61}$$

Upon examination of the variances, it is clear that they are not the same model. The point of this demonstration is to reinforce the principle that models should be a reflection

of reality. If the reality of the situation suggests Example 1, then use it. But if reality suggests Example 2, do not use Example 1 as a shortcut for representing a Example 2 situation.

Many CERs are derived by fitting curves to historical data. This is normally done by the method of least squares, which may be linear or nonlinear. Once the regression equation is accepted, the coefficients in that equation (which are really estimates themselves) are taken as constants and the independent variables in the equation are then treated as random variables.

Consider then a model of the general form

$$X = f(F_1, F_2, \ldots, F_q) \tag{62}$$

where f is a function of the factors $F_1, \ldots, F_q$. If $F_1, F_2, \ldots, F_q$ are random variables, X is also a random variable with expected value

$$E[X] = E[f(F_1, F_2, \ldots, F_q)] \tag{63}$$

and variance

$$Var[X] = Var[f(F_1, F_2, \ldots, F_q)] \tag{64}$$

The problem is then to find the expected value and variance of f, which may be a rather complicated function.

Two commonly used forms for the function f are

$$\text{Form 1} \qquad f(F_1) = a + bF_1^{E_1} \tag{65}$$

and

$$\text{Form 2} \qquad f(F_1, F_2, \ldots F_q) = bF_1^{E_1} F_2^{E_2} \ldots F_q^{E_q} \tag{66}$$

where a, b, and $E_1 \ldots E_q$ are parameters. One can represent the function f by a Taylor series expansion about the point $P_\mu = (\mu_1, \mu_2, \ldots, \mu_q)$ where μ_i is the expected value of F_i and apply equations (10) and (11) to approximate the expected value and variance of the function.

Using transforms, evaluation of Form 2 is quite direct but rather involved in the case of Form 1, which can be re-written as

$$f(F_1) = a+Z \tag{67}$$

where

$$Z = bF_1^{E_1}$$

The product $bF_1^{E_1}$ is evaluated using the Mellin transform which is

$$M[g(Z)] = b^{s-1} M_{F_1}[E_1 s - E_1 + 1] \tag{68}$$

In words, the Mellin transform of the distribution of Z, g(z), is b^{s-1} times the Mellin transform of the distribution of F_1 with $(E_1 s - E_1 + 1)$ substituted for the complex variable s. This transform M[g(z)] is then inverted yielding g(z). The Laplace transform is then used to evaluate $f(F_1)$ giving

$$L[h(f(F_1))] = e^{-sa} L[g(Z)] \tag{69}$$

The inversion process then yields the distribution $h(f(F_1))$ which is zero to the left of the point $f(F_1) = a$.

Form 2 is addressed using only the Mellin transform which is

$$M[h(f(F_1, F_2, \ldots, F_q))] = b^{s-1} M_{F_1}[E_1 s - E_1 + 1]\, M_{F_2}[E_2 s - E_2 + 1] \cdots M_{F_q}[E_q s - E_q + 1] \tag{70}$$

As before, the transform can be inverted for the distribution of $f(F_1,F_2,\ldots,F_q)$ or the necessary moments can be simply extracted.

This section has dealt with the modeling of O&S costs. The O&S cost modeling structure is summarized in Figure 13 . Given that the cost factors and, in turn, elements are random variables, it is now appropriate to discuss some possible distributions for these random variables.

$$\xrightarrow{\text{Factors } (+\ \&\ x)}$$

$$\begin{array}{lcl} F_1+F_2 & = & X_1 \\ & & + \\ F_3 x F_4 & = & X_2 \\ & & + \quad \Big\downarrow \text{ Elements } (+) \\ & & + \\ F_5F_6+F_7F_8+\ldots+F_q & = & X_m \end{array}$$

$$Y_1 + Y_2 + \ldots + Y_n = TOS$$

$$\xrightarrow[\text{Yearly Cost } (+)]{}$$

Figure 13: Modeling Structure

3.3 PROBABILITY DISTRIBUTIONS

A variety of probability distributions have been applied in cost modeling. Typically, the true distribution is unknown, necessitating some assumptions on the part of the analyst. One such assumption used in this research is that cost is continuous and that the factors used in arriving at cost are also continuous. In some cases, a continuous distribution may be used to approximate what is usually a discrete distribution. One example is manpower. In actual practice, one cannot meaningfully assign half a person to a particular base or weapons system, but it is felt that such minor distortion, when viewing the entire O&S cost picture, will be slight and, thus, can be reasonably disregarded. At this point, it seems reasonable to establish a set of criteria against which various candidate distributions can be judged. Wilder and Black [89:195] suggest the following:

1. Finite ends
2. Not necessarily symmetric
3. Unimodal
4. Computationally simple

To this list can be added the following:

1. Wide variety of shapes
2. Parameters easily determined
3. Conducive to subjective input

Not all the proposed distributions meet all of these criteria. Since little data and information exists regarding the actual distribution of O&S cost factors and elements, it is often necessary for the analyst to make assumptions regarding the distribution to be used. Such assumptions should reflect the reality of the modeling situation and make good intuitive sense. The choice will often be driven by the information available. For instance, if one can only reasonably determine the first two moments, it does not seem reasonable to use a distribution requiring, say, four moments for parameterization.

In many instances,the parameter values of the various distributions must be determined from subjective input. This is particularly true for new proposed systems where substantive data is lacking. Most often these subjective inputs are based on the opinion of experts, either collectively or individually. In collecting such data, it is desirable to keep the information required simple and to a minimum. This forces the analyst to make some basic assumptions and, in some cases, to use some simplifying approximations. The most basic assumption is which distribution should be used to model a certain factor, cost element, or situation. Once the distribution has been chosen, the analyst must obtain some information regarding the factor or cost element. In keeping the information on the cost factor

to a minimum, it will be assumed that the expert can furnish only a low estimate (L), most likely estimate (M), and high estimate (H). With this limited information in hand, the analyst must then determine applicable distribution parameter values.

Six candidate distributions are discussed in this section. This is by no means an exhaustive list, but these distributions have found past application in cost analysis. In most cases, however, they do not meet all the criteria listed above. These distributions are the normal, log normal, triangular, beta, rectangular, and gamma. The discrete Poisson distribution is also presented for use in computing random sums of random variables.

3.3.1 The Normal Distribution

The normal is perhaps the best known and most used distribution. It has found wide application and applies when small deviations from the mean are more frequent than large deviations and when positive and negative deviations of the same size are equally likely. This distribution is fully parameterized in terms of its mean and variance.

Mathematically, the normal probability distribution is defined by the probability density function

$$f(x) = (\sqrt{2\pi}\sigma)^{-1} \exp[-1/2[(x-\ )/\sigma]^2], \quad -\infty \leq x \leq \infty \qquad (71)$$

where

π = a constant (approximately 3.1416)

μ = expected value of x

σ = standard deviation of x

One specific variation of the normal distribution, the standardized normal is of particular interest. This variation, often referred to as the unit normal or z distribution, has a mean of zero and a variance of one and is related to the normal through the standardization formula

$$z = (x-\mu)/\sigma \tag{72}$$

Several cautions are in order regarding the use of this distribution. First, unfortunately, a closed form expression of the cumulative distribution function[18] does not exist and its evaluation can only be obtained through approximate numerical procedures. Second, it fails to meet several of the criteria set forth earlier. It does not have finite ends. In using this distribtion, one is automatically assuming that costs can go to infinity in both the positive and negative directions. Also, the distribution is symmetric about the mean, thus severely limiting the shapes it can attain.

In spite of its deficiencies, in the absence of any real indication to the contrary, this may be the best distribution to use in cost analysis, partcularly when one is work-

---

[18] The cumulative distribution function is defined as $F(X) = \int_{-\infty}^{X} f(t)\,dt$.

ing with the sums and products of random variables and mean and variance is all that is known.

In the case of the normal distribution, parameter values determined from subjective input must be approximated. An approximation can be made based on the fact that ninety-nine percent the observations fall within plus or minus three standard deviations of the mean. Thus, knowledge of L and H is all that is required. If one can obtain a range, L to H, of the random variable, then (L+H)/2 is the mean[19] and (H-L)/6 is the approximate standard deviation.

Before proceeding with this discussion of probability distributions, a few words concerning the central limit theorem are in order.

3.3.1.1 Central Limit Theorem

It can be shown that the sums of independent normally distributed random variables are themselves normally distributed and, that even if the random variables are not normally distributed, the distribution of the sum still tends to be normal. In a general form, relaxing the identically distributed requirement, Hillier and Lieberman [47:366] state that if the random variables $X_1, \ldots, X_n$ are independent with means $\mu_1, \ldots, \mu_n$ and variances $\sigma_1^2, \ldots, \sigma_n^2$ respectively, then

[19] For the normal distribution, the mean and mode or most likely are the same. The mean computed here can be compared to the M obtained subjectively in evaluating the appropriateness normality assumption.

the random variable Z_n,

$$Z_n = \left(\sum_n X_i - \sum_n \mu_i\right) / \sqrt{\sum_n \sigma_i^2} \qquad (73)$$

under certain regularity conditions is approximately normally distributed with zero mean and unit variance in the sense that

$$\lim_{n \to \infty} P(Z_n \leq b) = \int_{-\infty}^{b} (2\pi)^{-1} \exp(-Y^2/2)\, dY \qquad (74)$$

A number of central limit theorems for dependent random variables have been developed, although it is not possible to state a simple general result in this case [46:26-27].

In spite of the generalities of the central limit theorem, normality may or may not be a good representation of the sums of random variables involved in O&S costing. According to McNichols [71:119], the applicability of normality will depend on the number of variables in the sum, the relative shapes and spread of the distributions, and the degree of dependency, if any. He concludes that arbitrarily restricting the distribution of the sum to be normal could grossly understate the true risk. However, in the absence of more definitive information, the assumption of normality is both logical and warrented.

3.3.2 The Log Normal Distribution

The log normal distribution offers some improvement over the normal in that it has a finite left tail anchored at the origin and an infinite right tail, yet, it is not quite as easily handled and is limited in shape. The log normal density function is

$$f(t) = (\sigma\ t\ \sqrt{2\pi})^{-1} \exp[-1/2((\ln\ t-\mu)/\sigma\ )^{2}]\ ,$$

$$t \quad 0 \tag{75}$$

where μ and σ are parameters such that $-\infty < \mu < \infty$ and ln stands for natural logarithm.

If a random variable X is defined as $X = \ln t$, then X is normally distributed with mean μ and standard deviation σ. Using this substitution and the normal distribution, it can be shown that the mean and variance of the log normal distribution are

$$E[t] = \exp[\mu + \sigma^2/2] \tag{76}$$

and

$$Var[t] = \exp[2\mu + \sigma^2](\exp[\sigma^2]-1) \tag{77}$$

Values for the parameters μ and σ are determined from subjective input in the same way that these parameters are determined for the normal distribution.

3.3.3 The Triangular Distribution

The triangular distribution is relatively simple and is completely parameterized with knowledge of only three points. The probability density function is given by

$$f(x) = \begin{cases} [2(x-a)]/[(b-a)(m-a)] & \text{if } a \le x \le m \\ \\ [2(b-x)]/[(b-a)(b-m)] & \text{if } m \le x \le b \end{cases} \qquad (78)$$

where

a = lower limit (≥ 0)

m = most likely value (mode)

b = high limit

This distribution has all the properties listed. It is bounded and incorporates skewness. Further, it appears to be any easy concept to visualize and understand, especially for those not familiar with statistics [89:195].

The lone drawback to this distribution is the lack of smoothness at the point $x = m$. This necessitates the breaking of the transforms for the triangular distribution into two parts. That is,

$$L[f(x)] = \int_a^m e^{-sx} f_1(x)\ dx + \int_m^b e^{-sx} f_2(x)\ dx \qquad (79)$$

and

$$M[f(x)] = \int_a^m x^{s-1} f_1(x)\ dx + \int_m^b x^{s-1} f_2(x)\ dx \qquad (80)$$

where

$$f_1(x) = [2(x-a)]/[(b-a)(m-a)]$$

$$f_2(x) = [2(b-x)]/[(b-a)(b-m)]$$

For this distribution, knowledge of L, M, and H completely defines the distribution. Just set L = a, M = m, and H = b.

3.3.4 The Beta Distribution

The generalized beta distribution, along with its special cases, is probably the most used distribution in the area of stochastic cost estimation. It offers great versatility in terms of location and shape. The beta distibution probability density function defined on the interval [a,a+b] is

$$f(x) = [\Gamma(\alpha+\beta)/\Gamma(\alpha)\Gamma(\beta)\,b]\,((x-a)/b)^{\alpha-1} (1-(x-a)/b)^{\beta-1} \qquad (81)$$

where

$$\Gamma(\alpha) = \text{the gamma function} = \int_0^\infty y^{\alpha-1}\exp[-y]\,dy$$

and

$$a \le x \le a+b, \quad \alpha, \beta > 0$$

The parameter a is the low value, b the range, and a+b the high value of the distribution. The parameters α and β are shape parameters. A standardized version of the beta distribution[20] over the interval [0,1] is obtained by the transformation $y = (x-a)/b$ giving

$$f(y) = [y^{\alpha-1}(1-y)^{\beta-1}]/B(\alpha,\beta) \qquad (82)$$

[20] Sometimes referred to as a Type I beta as opposed to the generalized Type II.

where

$$B(\alpha,\beta) = \int_0^1 y^{\alpha-1}(1-y)^{\beta-1}dy = [\Gamma(\alpha)\Gamma(\beta)]/\Gamma(\alpha+\beta)$$

The cumulative distribution function of the standardized beta random variable is commonly called the incomplete B-function denoted by

$$F(y) = \int_0^Y [t^{\alpha-1}(1-t)^{\beta-1}]/B(\alpha,\beta)\ dt \qquad (83)$$

The beta distribution is particularly appealing to some in that it sets upper and lower limits on the cost factors and elements, it can have any left or right skewness, and it assumes many shapes.

Parameter estimation for the beta distribution is somewhat more involved since there are more parameters to estimate. Several subjective parameter estimation techniques have been proposed. The most well known is that developed as a part of the Program Evaluation and Review Technique (PERT). First used to model the distribution of activity durations in networks, the originators assumed that a low (L), high (H), and most likely (M) value for the time of each activity. Then setting the parameters a and b to a = L and b = H-L, the mean and variance are approximated by

$$\bar{X} = (L+4M+H)/6 \qquad (84)$$

$$S^2 = (H-L)^2/36 \qquad (85)$$

Since the original PERT proposal, it has been shown that this approximation corresponds to only three α,β combinations, thus severely limiting the shape and versatility of the distribution. The user should be aware of these limitations and possible errors prior to using the PERT approach.

As an alternative to the PERT approach, the procedure presented here was first introduced by Donaldson [30] and later modified by Coon [21]. It produces a more general class of beta distribution than the PERT, but assumes that the distribution is tangential to the horizontal axis at each end. This has the effect of limiting α and β to values greater than or equal to two.

From the previous discussion of the beta distribution and using the three estimates (L, M, and H), it can be shown that

$$L = \text{low} = a \tag{86}$$

$$H = \text{high} = a+b \tag{87}$$

$$M = \text{mode} = [a(\beta-1)+(a+b)(\alpha-1)]/(\alpha+\beta-2) \tag{88}$$

Substituting the above estimates and rearranging,

$$(H-M)/(M-L) = (\beta-1)/(\alpha-1) \tag{89}$$

This result is an 'assymetry' measure. The remainder of the derivation by Donaldson and Coon is somewhat involved, but the decision rules can be summarized as follows:

$$\begin{aligned} &\text{if } (H-M) > (M-L) \text{ set } \alpha=2 \text{ and } \beta=(H-M)/(M-L)+1 \\ &\text{if } (M-L) > (H-M) \text{ set } \beta=2 \text{ and } \alpha=(M-L)/(H-M)+1 \\ &\text{otherwise set } \alpha=\beta=2 \end{aligned} \tag{90}$$

Although this procedure does not allow a full range of the beta distribution, Donaldson has shown that it allows skew to range from -1.414 to +1.414, while the PERT is valid at only three values, -0.707, +0.707, and 0.0.

As another special case of the beta distribution, consider the rectangular distribution. It occurs when α and β are one.

3.3.4.1 The Rectangular Distribution

The rectangular distribution, often confused with the uniform distribution, is a continuous probability distribution which gives the probability that a sample value will be within a given interval when probability is directly proportional to the length of the interval. Mathematically, the rectangular distribution is defined by the probability density function

$$f(x) = 1/b \ , \ 0 \le a \le x \le a+b \qquad (91)$$

While this distribution does meet most of the above criteria, it is severely restricted with regard to shape. It is, however, easily parameterized with knowledge of L and H where b = H-L .

3.3.5 The Gamma Distribution

The gamma distribution, although well known and widely used, has found limited application in the field of cost analysis. Its probability density function is of the form

$$f(x) = (\Gamma(\alpha))^{-1} \beta^{-\alpha} (x-k)^{\alpha-1} \exp[-(x-k)/\beta] \tag{92}$$

with

$\alpha, \beta, k > 0$, $x \geq k$

where

α = shape parameter

β = scale parameter

k = location parameter

This distribution is characterized by a finite lower tail and infinite upper tail. With k equal to zero, the lower tail is anchored at the origin indicating that the lower bound on cost is zero and the upper bound is infinity. Such a distribution could have a rather large variance, but may be indicative of the true risk involved in some programs. Even if k is not zero, this distribution reflects the risk of escalating costs and budget overruns which seem to be so common in weapons system acquisition and support. Others argue that there is an upper limit on affordability, but such a limit is neither clearly defined nor does it seem to be stationary.

In reference to the previously defined criteria, this distribution meets all with the exception noted above. It

should be noted, however, that the gamma distribution is limited to positive skewness.

Parameter estimation for the gamma distribution suffers the same general limitations as the beta distribution due to the form of the Incomplete Gamma function. Therefore, approximations for the mean and variance will be used to estimate the parameters. The procedure presented was developed by Perry and Greig [86] and requires that L and H be redefined as L_δ and H_δ where L_δ is a low estimate such that the probability of anteceding it is δ and H_δ is a high estimate such that the probability of exceeding it is also δ. With δ=.05, Perry and Greig have shown that

$$\mu \doteq (L_\delta + .95M + H_\delta)/2.95 \tag{93}$$

and

$$\sigma^2 \doteq ((H_\delta - L_\delta)/3.25)^2 \tag{94}$$

From the properties of the gamma distribution, it can be shown that

$$\mu = \text{mean} = \alpha\beta + k \tag{95}$$

$$\sigma^2 = \text{variance} = \alpha\beta^2 \tag{96}$$

$$M = \text{mode} = \beta(\alpha - 1) + k \tag{97}$$

Subtracting (97) from (95),

$$\beta = \mu - M \tag{98}$$

where the value of μ is determined from (93) above. Equation (96), with σ^2 determined from equation (94), is then used to solve for α. With α and β both known, equation (95)

is then used to solve for k. Thus, the gamma distribution parameters can be determined using the three subjective inputs.

3.3.6 The Poisson Distribution

Before concluding this section, it might be helpful to include a discrete distribution to be used in addressing the random sums of random variables. For this purpose, the Poisson distribution has been selected. Its probability mass function is

$$p(n) = \lambda^n/n! \; e^{-\lambda} \quad , \; n=1,\ldots,\infty \tag{99}$$

where λ is the lone parameter equal to the mean and variance of the distribution.

Haight [42:12] has shown that the Poisson distribution has a unique mode satisfying the relationship

$$\lambda-1 \leq \text{mode} \leq \lambda \tag{100}$$

Therefore, one can use the subjective estimate of the mode, M, as an approximation for λ. If the analysis is sensitive to this parameter, both extremes in (100) can be used.

Table 10 summarizes the results of this section and gives some pertinent information relating to transforms.

TABLE 10

Probability Distribution Summary

Name	Density Function	Mean	Variance	Mode	Laplace Transform	Mellin Transform
Normal	$\frac{1}{\sqrt{2\pi}\sigma} e^{-1/2((x-\mu)/\sigma)^2}$ $-\infty \le x \le \infty$ Standard Normal $\frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ where $z=(x-\mu)/\sigma$	μ	σ^2	u		
Log Normal	$\frac{1}{\sigma t\sqrt{2\pi}} e^{-1/2((\ln t-\mu)/\sigma)^2}$ $t \ge 0$	$e^{\mu+\sigma^2/2}$	$e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$	$e^{\mu-\sigma^2}$		
Triangular	$\frac{2(x-a)}{(b-a)(m-a)}$ if a x m $\frac{2(b-x)}{(b-a)(b-m)}$ if m x b	Appendix D	Appendix D	m	Appendix D	Appendix D
Beta	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)\,b}\left(\frac{x-a}{b}\right)^{\alpha-1}\left(1-\frac{x-a}{b}\right)^{\beta-1}$ $a \le x \le a+b,\ \alpha,\beta \ge 0$	$a+\frac{b\alpha}{\alpha+\beta}$	$\frac{b^2\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$	$\frac{(\alpha-1)(a+b)+(\beta-1)a}{\alpha+\beta-2}$		
	Standard Beta $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}$ where $y=(x-a)/b$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$	$\frac{\alpha-1}{\alpha+\beta-2}$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)}\sum_{j=0}^{\infty}(-1)^j\frac{\Gamma(\alpha+j)s^j}{\Gamma(\alpha+\beta+j)\Gamma(j+1)}$	$\frac{\Gamma(\alpha+\beta)\Gamma(s+\alpha-1)}{\Gamma(\alpha)\Gamma(s+\alpha-1+\beta)}$
Rectangular	$\frac{1}{b}$ $a \le x \le a+b$	$\frac{2a+b}{2}$	$\frac{b^2}{12}$		$\frac{1}{sb}\left(e^{-as}-e^{-(a+b)s}\right)$	$\frac{(a+b)^s-a^s}{sb}$
Gamma	$\frac{1}{\Gamma(\alpha)}\ \beta^{-\alpha}(x-k)^{\alpha-1}e^{-(x-k)/\beta}$ $\alpha,\beta,k \ge 0 \quad x \ge k$	$\alpha\beta+k$	$\alpha\beta^2$	$\beta(\alpha-1)+k$	if $y=x-k$ $(1+\beta s)^{-\alpha}$	if $y=x-k$ $\beta^{s-1}\frac{\Gamma(s+\alpha-1)}{\Gamma(\alpha)}$

Name	Mass Function	Mean	Variance	Mode	Geometric Transform
Poisson	$\frac{\lambda^n}{n!}e^{-\lambda}$ $n=1,\ldots,$	λ	λ	$\lambda-1 \le m \le \lambda$	$e^{-\lambda(1-z)}$

3.4 PRESENTING THE RISK ANALYSIS

Once the risk analysis has been accomplished, the results must be meaningfully conveyed to decision makers. In light of the objections stated in Chapter 1, this is no easy task. In meeting the needs of decision makers, the information presented must be clear, concise, easily understood and relevant to the decision.

Perhaps, the most simple and direct statement of risk is variance. Unfortunately, the mere expression of variance has little meaning or intuitive appeal to most decision makers. Numbers must be coupled with a visual sense of magnitude. One way to do this is through a mean-variance plot.

In comparing alternatives using a mean-variance plot, the mean is plotted on the horizontal axis and the variance on the vertical axis. (See Figure 14 .) Each alternative is represented by a point on the plot. In comparing alternatives, low mean cost is preferred to high and low variance (risk) is also preferred to high. Thus, in the case of alternatives A and C, both with equal mean, A is preferred to C as it has least variance. Likewise, in the case of alternatives A and B, both with equal variance, A is preferred to B since it has the lower mean cost. Therefore, it can be said that A is preferred to all other alternatives including D. A problem of choice occurs if B and C are the only competing options. Alternative B has lower variance but higher

mean than C. This is where decision makers' preference for risk invades the decision process. A risk averse person may choose B, while a risk seeking person may choose C. The decision may also be influenced by the distance which separates the points on the plot. As a variation, the mode can be substituted for the mean and standard deviation for variance.

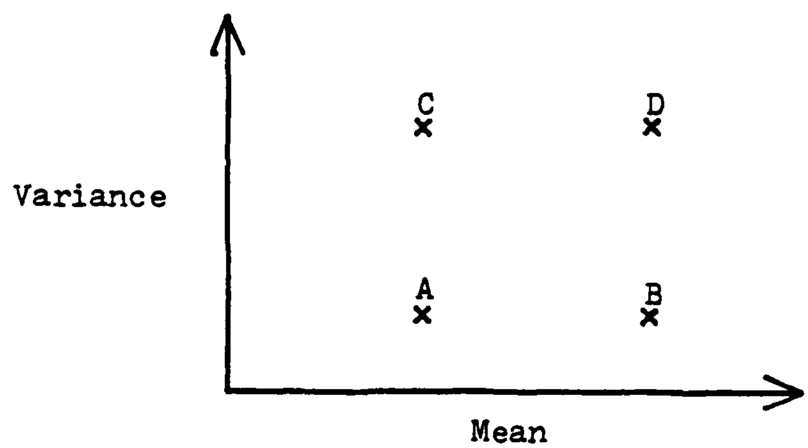


Figure 14: Mean-Variance Plot

Tolerance intervals can also be used in presenting the risk analysis results. A statistical tolerance interval is an interval within which one can state with a given probability of being correct that at least a prespecified proportion of a distribution is located [41:77]. The end

points of such intervals are called tolerance limits. Tolerance intervals are not to be confused with confidence intervals. A confidence interval defines limits which will cover or contain a population parameter with a certain confidence. Although this difference may seem subtle, tolerance intervals and confidence intervals answer quite different questions.

In order to apply tolerance limits all explainable causes of variability must be detected and eliminated and certain assumptions must be made concerning the population under study. If one assumes that the parent population is normally distributed with known sample mean and variance, the tolerance limits take the form

$$\bar{X} \pm Ks \tag{101}$$

with K chosen so that one may expect these limits to include at least P percent of the population at some prescribed probability level γ. Standard deviation is represented by s. Table 11 lists K factors for various values of P assuming an infinite sample size and point nine five (.95) probability level [83:103]. The reader will note that with an infinite sample size the K factors are the same as the standard normal random variable.

Computing, plotting, and using the cumulative distribution function is also a convenient and descriptive way of presenting a risk analysis. By definition, the cumulative

TABLE 11

Two-Sided Tolerance Factors

P	K
.75	1.150
.90	1.645
.95	1.960
.99	2.576

distribution function is a function that describes the probability that the random variable X will take on a value less than or equal to a prespecified value X within its domain. Mathematically, this is

$$F(b) = P(X \leq b) = \int_{-\infty}^{b} f(X)\ dX \tag{102}$$

where f(X) is the probability density function.

Thus, the cumulative distribution function allows the analyst to make probability statements regarding costs of weapons systems. For instance, refering to Figure 15 , it can be said that there is a probability of .75 that the cost will be less than or equal to $5.1 million or that there is a .50 probability that it will be between $2.8 and $5.1 million. Competing systems can also be compared by plotting their cumulative distributions on the same axis.

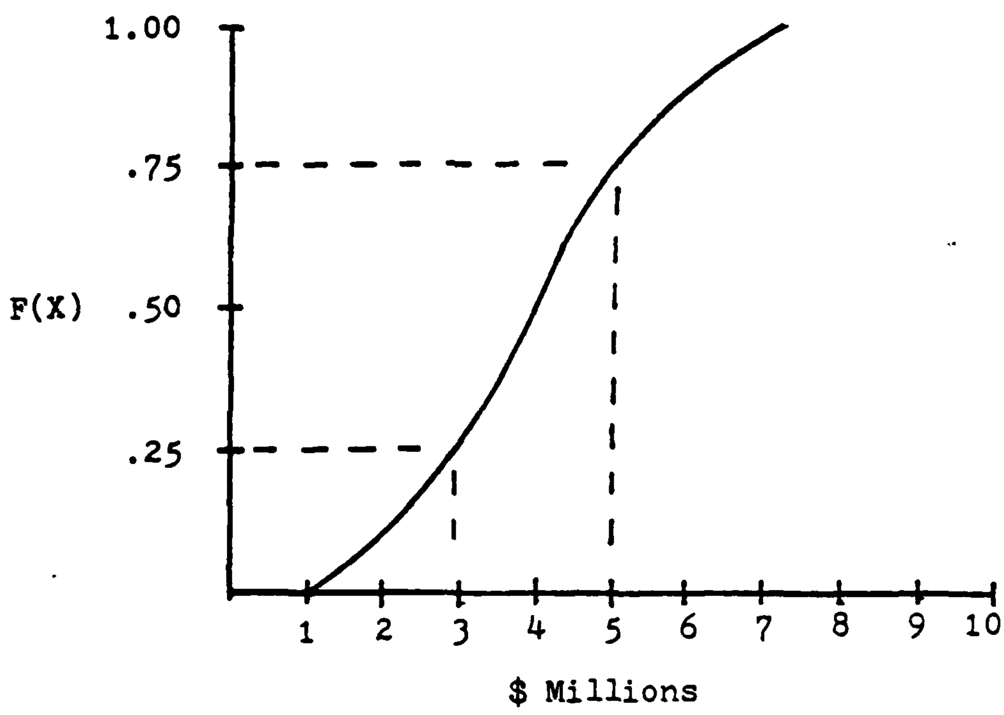


Figure 15: Cumulative Distrbution Function Plot

Yet another way to present risk analysis results, in conjunction with the cumulative distribution function, is the floating bar graph. To construct the graph, take the middle ninety percent of each distribution. That is,

$$P(X_{.05} \leq X \leq X_{.95}) = .90 \tag{103}$$

Plot this range on a bar graph as shown in Figure 16 . The graph can be enhanced by indicating the mean and modal cost for each alternative. This presentation indicates the vari-

ability plus the location of the mean and mode for decision makers.

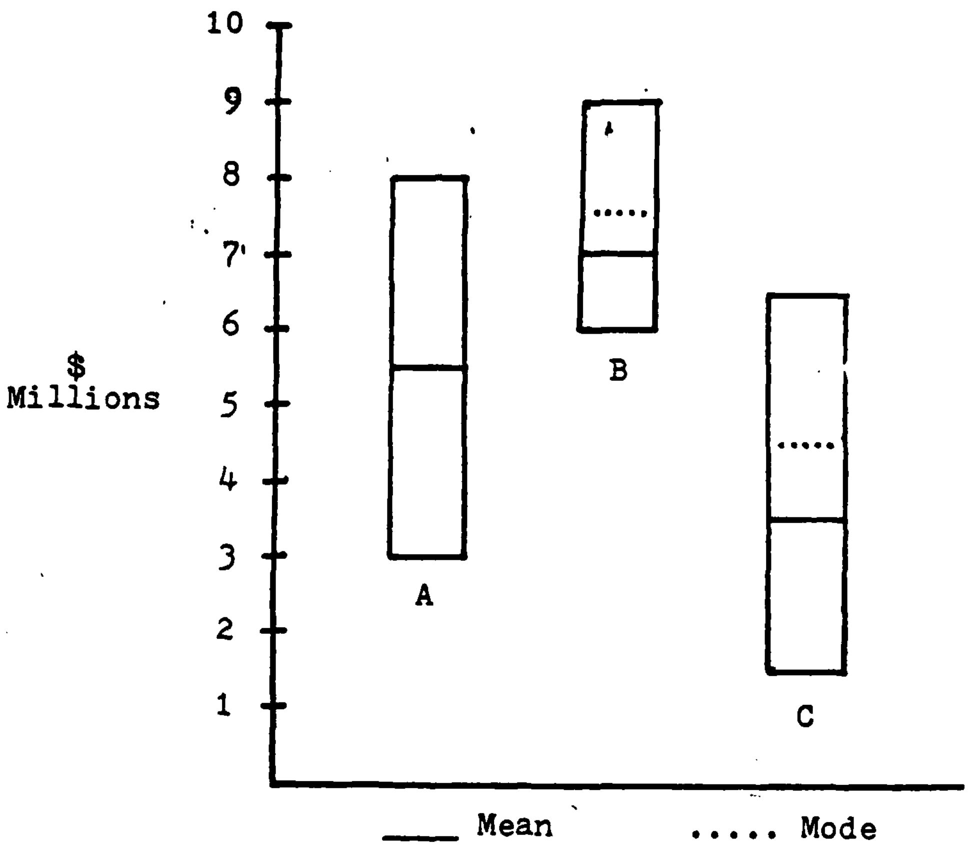


Figure 16: Floating Bar Graph

From this short discussion, it is evident that risk analysis results can be presented to decision makers in a meaningful, informative way. The next chapter demonstrates and presents examples of the material discussed in this chapter.

Chapter IV

APPLICATIONS

The purpose of this chapter is to demonstrate some applications of the theoretical aspects of risk analysis presented in the previous chapter. Once again the emphasis is on operating and support (O&S) costs. The specific modeling methods and probability distributions used are for illustrative purposes only and are not intended to imply that others could not be used. The objective is to demonstrate as many of the techniques presented in the previous chapter as possible within a given, typical O&S cost analysis scenario.

4.1 THE GENERAL APPROACH

A four step approach is recommended in performing any cost risk analysis. These steps include:

1. Determine most likely point estimate
2. Perform preliminary risk analysis on cost drivers
3. Accomplish indepth risk analysis where warranted
4. Do sensitivity analysis

The first step is a preliminary analysis to determine the cost drivers and satisfy the need for a point estimate.

This estimate should be based on the most likely cost of each element and need only be done for a single, typical year in order to identify the cost drivers. In doing the point estimate, the analyst must determine the appropriate estimating technique to be used for each cost element.

With the cost drivers identified, the next step is to perform a preliminary risk analysis. Additional information is gathered on the parameters relating to the cost drivers which will permit the analyst to do a preliminary risk analysis. This analysis is done only on the cost drivers or cost elements where risk is of concern and should be done using the sums and products of random variables method. These results will indicate if further, indepth risk analysis is necessary.

Indepth risk analysis is then performed on parameters or cost elements which warrant such attention. This 'microscopic' analysis can be done using transforms to determine the type and/or moments of the distributions involved. Information gleaned from this step can be used to update and refine the preliminary risk analysis.

The last step is a sensitivity analysis on those parameters or cost elements which are shown to be quite sensitive to the analysis.

As a modified approach, the analyst may determine the cost drivers a priori either from personal experience or

from the program scenario. In this way, data needed for the risk analysis can be collected along with ths information necessary to do the initial point estimate. This modified approach may save some time and effort. For new systems, the cost drivers can often be identified by examining costs and cost related parameters on existing, related systems.

Since much of the data relating to new, perspective systems is based on expert opinion, a few words and cautions on collecting such data are now in order.

4.2 SUBJECTIVE INPUTS

Most experts, when making subjective estimates, attempt to compare the situation confronting them to past, related situations which they have encountered. This is done directly or indirectly; overtly or covertly. Some are better at using their experience than others, not necessarily because their experience is of a higher quality but because of the flexibility they display in using their experience. It is the task of the analyst to lend assistance and direction in this process.

Probably the most difficult task confronting experts is distinguishing between what they want to happen and what they think will actually happen. Often the expert has a vested interest in the area for which information is sought and may assume somewhat of an advocate role. If the expert

is personally involved with a program, that person will naturally want the program to succeed and, unless great care is shown, there is a tendency to be less objective than otherwise. Further, if the estimate provided will also be used as a yardstick against which the expert's performance is measured then there is a natural tendency to be conservative.

Often a group, rather than a single expert, is used. It can be argued that this is one way of overcoming some of the previously mentioned biases. The problem which then arises is that of combining a number of different estimates into a single estimate. Should an average be used or should more weight be given to the estimates of more knowledgeable experts? If there is a group discussion to resolve differences, there is a danger of the group being unduly influenced by one or two dominant personalities.

One approach to group forecasting which has received a great deal of attention in the literature is the Delphi technique. This technique provides a means to incorporate feedback into the estimating process. It involves interrogating experts by means of a sequence of questionaires; the first of which is an independent input from each expert. These inputs are then aggregated and a summary is then presented to each expert who is then asked to reconsider the previous, independent input in light of the new information

contained in the summary. The process is then repeated until consensus is achieved. Thus, through this technique, personal contact and domination by forceful personalities is avoided.

Apart from enabling a more meaningful analysis, estimating risk can have distinct advantages. Experts are often far more willing to make uncertainty estimates than point estimates since point estimates may seem like a personal commitment on the part of the experts.

In gathering subjective estimates, the problem and information needed must be clearly defined. All possible biases should be eliminated. To avoid anchoring, the experts' attention should be initially directed toward the extreme values. Thus, the high and low estimates should be obtained before the the most likely estimate. The analyst must first allow the expert to appreciate the full range of risk. Questions such as, 'How bad could the variable get?' and 'Could the value in fact turn out to be better than anticipated?' should be asked. A dialogue between the analyst and expert must be established with the analyst being careful not to lead the expert.

Once the high and low values have been reasonably determined, the expert is asked to give a best guess or most likely value for the variable. As a final test, the analyst should assess the expert's willingness to place a bet on the

value of the variable being greater than the high or less than the low estimate. This often leads to a revision of the estimates.

The gathering of subjective estimates is an art and requires the analyst to use sound judgement, ingenuity, perseverence, and good human relations.

4.3 THE SCENARIO

The Air Force is conducting preliminary cost studies on the Follow-on-Fighter (FoF) which will augment the current fleet of F-15 and F-16 aircraft beginning in 1995. The FoF is to be used in an air superiority role, thereby freeing current aircraft for interdiction and close air support roles. Three squadrons of twenty-four aircraft each will be required with all production to take place in 1995. The operational life of this aircraft is expected to be only six years due to the introduction of the Space Defense System, which will be operational in the year 2000.

Industry has proposed two versions of the FoF: the XF-1 and the XF-2. The XF-1 will be a single engine, single seat aircraft, whereas, the XF-2 will be dual engine, dual seat. Both will have the same operational capability, but with several additional basic differences. Due to weight and speed specifications, it will be necessary to build the XF-1 airframe from new, light weight, composite materials. The

development and production of these new materials is considered to be high risk. In contrast, the XF-2 will have a more conventional construction, but will exceed the XF-1 in both size and weight.

The XF-1 will be powered by the new astrojet engine which will use jet fuel impregnated with ether. The ether impregnated fuel will produce more thrust, but, on the negative side, more heat, than conventional fuel. Due to the unproven technology, the astrojet engine is also considered to be high risk. The XF-2 will be powered by conventional jet engines.

The XF-1 avionics package is also a concern. Due to limitations on its single crewperson, the XF-1 avionics must operate automatically, as opposed to the more manual XF-2 avionics. Due to the automated features, the XF-1 avionics will challenge the state of the art and, therefore, presents a high risk.

The FoF O&S cost estimate is to be done in accordance with the CAIG-approved O&S cost element structure (CES) and is to include a risk analysis.

4.4 MOST LIKELY POINT ESTIMATE

As the first step in the analysis, a preliminary point estimate using most likely values and appropriate estimating techniques was accomplished. This is a single year steady state estimate. The results of this analysis are presented in Table 12 .

TABLE 12

FoF Preliminary Cost Analysis

$millions FY83

Category	XF-1	XF-2
Unit Mission Personnel	31.195	34.111
Unit Level Consumption	35.123	39.233
Depot Level Maintenance	18.282	17.947
Sustaining Investment	11.066	14.798
Installation Support Personnel	7.450	6.880
Indirect Personnel Support	8.220	8.240
Depot Non-Maintenance	2.080	2.164
Personnel Acquisition & Training	.000	.000
Total	113.416	123.373

From Table 12, it is readily apparent that Unit Mission Personnel, Unit Level Consumption, Depot Level Maintenance, and Sustaining Investment are the FoF cost drivers. These four elements alone account for about eighty-five percent of the total O&S cost.

Upon closer examimnation, the analyst identified maintenance personnel as a subelement under Unit Mission Personnel for risk analysis, particularly in the case of the XF-1 with its new engine and avionics. Fuel consumption is the major contributor to Unit Level Consumption and is therefore identified for risk analysis. Depot Maintenance, although difficult to estimate, is the third highest cost element for both aircraft and is, therefore, subject to risk analysis. Under Sustaining Investment, replenishment spares is the major contributor to cost, particularly in the areas of avionics and engines. Therefore, these areas deserve special attention.

The costs identified for risk analysis are consistent with the scenario and represent most of the risk in this new program. They account for sixty-nine percent and seventy-five percent of the total support cost for the XF-1 and XF-2 respectively.

4.5 PRELIMINARY RISK ANALYSIS

4.5.1 Yearly Cost Computation

4.5.1.1 Maintenance Personnel

The analyst has determined that analogy using an F-16 baseline for the XF-1 and an F-15 baseline for the XF-2 is the most reasonable approach to maintenance personnel cost risk analysis. The first task is then to determine the number of personnel necessary to support the FoP. Table 13 lists the necessary information.

TABLE 13

Maintenance Personnel Requirements

	F-16 Baseline	XF-1 Additions
Total	1728	35
Avionics	197	30
Engines	118	5
	F-15 Baseline	**XF-2 Additions**
Total	1774	10
Avionics	251	7
Engines	192	3

Baseline Source: Headquarters, Tactical Air Command

In consulting with Air Force personnel experts regarding the PoP additional maintenance personnel, the analyst surmizes that the additional personnel follow a Poisson distribution with mean, mode, and variance equal to 35 and 10 for the XF-1 and XF-2 respectively. Thus, using an adaptation of equation (49), the number of maintenance personnel (MP) is represented by the equation

$$MP = Base + Change = W + Z \qquad (104)$$

and

$$E[MP] = W + E[Z] \qquad (105)$$

$$Var[MP] = Var[Z] \qquad (106)$$

These calculations are summarized in Table 14 .

TABLE 14

Maintenance Personnel Calculation Summary

	XF-1	XF-2
E[MP]	1763	1784
Var[MP]	35	10

The next step is to determine the pay for these maintenance personnel, which is summarized in Table 15 .

Now combining the results of Tables 14 and 15 and using the additive model where the number of terms in the sum is a

TABLE 15

Maintenance Personnel Pay

Grade	Composite Pay FY83[a]	Members in Grade[b]	Fraction of Force
E-9	$35,285	4,749	.01
E-8	29,859	9,478	.02
E-7	25,756	34,402	.08
E-6	21,876	53,344	.12
E-5	18,346	102,261	.22
E-4	15,497	102,269	.22
E-3	12,618	120,082	.27
E-2	11,440	26,244	.06
			1.00

E[PAY] = $17,513

Var[PAY] = 23,703,700

a - Source: AFR 173-13
b - Source: Air Force Magazine 66 (May 1983): 165

Note: $E[Y] = \sum YP(Y)$

$Var[Y] = \sum (Y-E[Y])^2 P(Y)$

random variable, application of equations (27) and (28) yield the results shown in Table 16 with detailed calcula-

tions for the XF-1 contained in Appendix E. The symbol 'SD' stands for standard deviation which is, of course, the square root of the variance. In later parts of the paper, the symbol 'EE' is used to represent scientific notation and means to multiply the number preceeding the 'EE' by ten to the power of the number following the 'EE'.

TABLE 16

Maintenance Personnel Yearly Pay Summary

$millions FY83

	XF-1	XF-2
$E[X_1]$	30.875	31.243
$SD[X_1]$	0.229	0.213

4.5.1.2 Fuel

The analyst chose the factor based cost estimating relationship (CER) for fuel from the Logistics Support Cost (LSC) Model [65] to estimate the FoF fuel consumption. The CER, in a slightly simplified form, is

$$Z = EPA \times FH \times FC \times FR$$

$$= CF_1F_2F_3 \qquad (107)$$

where

Z = Yearly fuel cost for a single aircraft

EPA = Number of engines per aircraft (C)

FH = Flying hours per aircraft per year (F_1)

FC = Fuel cost per gallon (F_2)

FR = Fuel consumption rate of one engine in gallons per flying hour (F_3)

After consulting with various experts in the areas of engine and fuel technology, the analyst was able to determine values for those variables as shown in Table 17 . The mean and variance were estimated using equations (84) and (85) respectively. Although the assumption of a beta distribution is an arbitrary one, it has served its purpose of locating the expected value with respect to L, M, and H in what seems to be a reasonable way. The calculations are simple and straightforward and all available information is used. In the absence of any information to the contrary, this seems like a reasonable assumption at this point in the analysis.

The reader will immediately note the higher fuel cost for the XF-1 due to the added ether and the greater variance in fuel rate for this aircraft due to its unproven technology. The analyst used a Taylor series expansion of the CER to estimate its expected value and variance by applying equations (9), (10), and (11) with the following results:

$$E[Z] = CF_1F_2F_3|_{P_\mu} \tag{108}$$

$$\mathrm{Var}[Z] = (\partial Z/\partial F_1|_{P_\mu})^2\sigma_{F_1}^2 + (\partial Z/\partial F_2|_{P_\mu})^2\sigma_{F_2}^2 + (\partial Z/\partial F_3|_{P_\mu})^2\sigma_{F_3}^2 \tag{109}$$

TABLE 17

Fuel CER Variable Values

Name	Var	Acft	L	M	H	Mean	Variance
FH	F_1	Both	336	360	408	364	144
FC	F_2	XF-1	1.95	2.00	2.10	2.00	.0006
		XF-2	1.30	1.34	1.38	1.34	.0002
FR	F_3	XF-1	490	550	655	558	756
		XF-2	495	500	510	501	6.25

EPA is a constant equal to one for the XF-1 and two for the XF-2.

where

$$\partial Z/\partial F_1 = CF_2F_3$$

$$\partial Z/\partial F_2 = CF_1F_3$$

$$\partial Z/\partial F_3 = CF_1F_2$$

Substituting the appropriate values from Table 17, it can be shown that for the XF-1,

$$E[Z] = 406{,}224 \qquad (110)$$

$$Var[Z] = 604{,}766{,}259 \qquad (111)$$

and for the XF-2

$$E[Z] = 488{,}736 \qquad (112)$$

$$Var[Z] = 292{,}154{,}748 \qquad (113)$$

The additive model was then used to compute the yearly fuel cost for the fleet of 72 aircraft. The results are summa-

rized in Table 18 with detailed calculations for the XF-1 in Appendix F.

TABLE 18

Yearly Fuel Cost Summary

$millions FY83

	XF-1	XF-2
$E[X_2]$	29.248	35.189
$SD[X_2]$	0.209	0.145

4.5.1.3 Depot Maintenance

The analyst found depot maintenance to be the most difficult cost element to estimate due to FoF risk and uncertainty in this area and a lack of good, substantive data. Therefore, pure analogy based on F-15 and F-16 baselines was used after consulting with Air Force logistics and depot maintenance experts. These costs are summarized in Table 19 .

Equation (49) was used as the basic model for estimating depot maintenance cost with Z remaining the same for each year. Equations (50) and (51) were used to estimate the expected value and variance with the results summarized in Table 20 . Equations (84) and (85) were used to estimate the

TABLE 19

Depot Maintenance Cost

$millions FY83

XF-1

	F-16 Baseline	Change L	Change M	Change H
Total Depot Maint	12.272	3.500	6.010	9.000
Periodic Maint	5.323	1.000	2.010	3.000
Engines	3.499	0.500	1.500	2.000
Avionics	0.460	2.000	2.500	4.000
Other	2.990			

XF-2

	F-15 Baseline	Change L	Change M	Change H
Total Depot Maint	14.610	1.000	3.337	4.050
Periodic Maint	2.928		0.312	
Engines	7.447	1.000	3.000	4.000
Avionics	2.475	0.000	0.025	0.050
Other	1.760			

Baseline Source - VAMOSC FY81 Report

mean and variance for Change (Z). The beta distribution was used for the reasons previously discussed.

4.5.1.4 Replenishment Spares

Replenishment spares are a continuing source of risk and uncertainty in Air Force O&S cost estimation. For this rea-

TABLE 20

Depot Maintenance Yearly Cost Summary

$millions FY83

	XP-1	XP-2
$E[X_3]$	18.362	17.680
$SD[X_3]$	0.001	0.001

son, a great deal of attention has been focused in this area in recent years. The analyst for the PoP has chosen regression based CER's from the Modular Life Cycle Cost Model (MLCCM) [75] to estimate the support cost for avionics and engines. Engine spares are addressed first.

4.5.1.5 Engine Spares

The CER, as it appears in the MLCCM, is

$$\$/FH(ENG) = (1.7031 \times 10^6)(ACTTYPE)^{2.4694}(AVTBOH)^{-1.6850}(MAXMACH)^{0.7974} \quad (114)$$

where

$/FH(ENG) = Cost per flying hour per engine

ACTTYPE = Aircraft type factor which is 1.0 for fighter aircraft

AVTBOH = Average time between engine overhaul

MAXMCH = Maximum mach number at optimum altitude

Restating in a more simple form,

$$Z = CF_4^{-1.6850}F_5^{0.7974} \tag{115}$$

where

$$C = (1.7031 \times 10^6)(ACTTYPE)^{2.4694}$$

$$F_4 = AVTBOH$$

$$F_5 = MAXMCH$$

Once more drawing in the Taylor series expansion and equations (10) and (11) and assuming independence of the variables concerned, it can be shown that

$$E[Z] = CF_4^{-1.6950}F_5^{0.7974}|_{P_\mu} \tag{116}$$

$$Var[Z] = (\partial Z/\partial F_4|_{P_\mu})^2\sigma_{F_5}^2 + (\partial Z/\partial F_5|_{P_\mu})^2\sigma_{F_4}^2 \tag{117}$$

where

$$\partial Z/\partial F_4 = -1.6850\ C\ F_4^{-2.6850}\ F_5^{0.7974}$$

$$\partial Z/\partial F_5 = 0.7974\ C\ F_4^{-1.6850}\ F_5^{-0.2026}$$

After consulting with development engineers and other engine experts including pilots, values for the independent variables were determined as listed in Table 21 .

The analyst must first compute the cost per flying hour per engine using the data from Table 21 and the expressions for expected value and variance above. Values for the XF-2 were multiplied by two in accordance with the formulae in Table 6 . Next, using the multiplcative model and Tables 7 and 17, the cost per aircraft per year was determined. Lastly, the cost for the 72 aircraft fleet was computed using the additive model. These results are summarized in Ta-

TABLE 21

Spare Engine CER Variable Values

Name	Var	Acft	L	M	H	Mean	Variance
AVTBOH	F_4	XF-1	150	225	350	233	1111
		XF-2	200	275	300	267	278
MAXMCH[a]	F_5	XF-1	1.95		2.05	2.00	.0008
		XF-2	2.00		2.15	2.075	.0019

a - A range only was supplied for MAXMCH indicating all values within the range have equal probability. Assumed a rectangular distribution with mean and variance computed in accordance with Table 10 .

ble 22 with detailed calculations for the XF-1 in Appendix G.

TABLE 22

Engine Spares Yearly Cost Summary

$millions FY83

	XF-1	XF-2
$E[X_4]$	7.967	13.052
$SD[X_4]$	0.229	0.171

4.5.1.6 Avionics Spares

Avionics spares were computed much the same as engine spares using the MLCCM regression based CER

$$\$/FH = (2.2956 \times 10^{-6})(LENSPN)^{0.8545}(UTLRAT)^{-0.1169}(AVNCWT)^{0.7979} \tag{118}$$

where

$/FH = Cost of avionics spares per flying hour

LENSPN = Aircraft length plus wing span in feet

UTLRAT = Flying hours per aircraft per year

AVNCWT = Avionics weight in pounds

Transforming the CER to a more useful form,

$$Z = CF_6^{0.8545}F_1^{-0.1169}F_7^{0.7979} \tag{119}$$

where

$C = (2.2956 \times 10^{-6})$

F_6 = LENSPN

F_1 = UTLRAT = FH

F_7 = AVNCWT

Appropriate values for the independent variables are contained in Table 23 .

Following the general procedure as that used for spare engines, the cost of avionics spares was computed. These results are summarized in Table 24 . Detailed calculations for the XF-1 are in Appendix H .

The yearly costs for those elements subject to risk analysis are summarized in Table 25 .

TABLE 23

Spare Avionics CER Variable Values

Name	Var	Acft	L	M	H	Mean	Variance
LENSPN[a]	F_6	XF-1	80		85	82.5	2.08
		XF-2	100		105	102.5	2.08
UTLRAT	F_1	Both	336	360	408	364	144
AVNCWT[a]	F_7	XF-1	2500		3000	2750	20833
		XF-2	1500		1700	1600	3333

a - A range only was supplied for LENSPN and AVNCWT indicating all values within the range have equal probability. Assumed a rectangular and mean and variance computed in accordance with Table 10 .

TABLE 24

Avionics Spares Yearly Cost Summary

$millions FY83

	XF-1	XF-2
$E[X_5]$	0.734	0.568
$SD[X_5]$	0.006	0.003

The equation for the expected value of the total yearly cost, Y, using the additive model is

$$E[Y] = C + E[X_1] + \dots + E[X_5] \qquad (120)$$

TABLE 25

Risk Analysis Yearly Cost Summary

$million FY83

Aircraft	Element	E[X]	SD[X]
XF-1	X_1	30.875	0.229
	X_2	29.248	0.209
	X_3	18.362	0.001
	X_4	7.967	0.229
	X_5	0.734	0.006
XF-2	X_1	31.243	0.213
	X_2	35.189	0.145
	X_3	17.680	0.001
	X_4	13.052	0.171
	X_5	0.568	0.003

where C is the most likely value of the sum of the cost elements not subjected to risk analysis. For the XF-1,

$$E[Y] = 35.310 + 87.186 = 122.496 \tag{121}$$

and for the XF-2,

$$E[Y] = 35.384 + 97.732 = 133.116 \tag{122}$$

Unfortunately, the variance is not so easily computed since three of the elements, fuel (X_2), engine spares (X_4), and avionics spares (X_5), are not independent due to the common factor, flying hours (F_1), used in their respective CER's. To compound the problem, the analyst's efforts to secure subjective estimates of the covariances from the var-

ious experts proved fruitless. Thus, the only recourse was to determine the correlation between fuel cost and replenishment spares for past and current related systems. This was done in Table 26 .

TABLE 26

Fuel/Spares Correlation

$millions FY82

Aircraft	Fuel	Spares[a]
F-4E	13.450	1.798
F-15	11.530	6.067
F-16	6.631	3.637
F-111A	9.736	8.002

r = correlation coefficient = -.23

a - Spares includes all spares. No separate breakdown for avionics spares and engine spares is available from the source used.

Source: AFR 173-13

Statistically, it is known that

$$\mathrm{Cov}[X_i, X_j] = r\ \mathrm{SD}[X_i]\ \mathrm{SD}[X_j] \qquad (123)$$

where

$\mathrm{Cov}[X_i, X_j]$ = Covariance of X_i and X_j

r = Correlation coefficient

$SD[X_i]$ = Standard deviation of X_i

Therefore, using the additive model in the dependent case,

$$Var[Y] = 1.2528EE11 \quad (124)$$

for the XF-1, and

$$Var[Y] = 8.3889EE10 \quad (125)$$

for the XF-2. The variances used in computing Var[Y] are listed in Appendix I . Next, the discounted expected cost and variance over the operational life of the system must be computed.

4.5.2 <u>Total Discounted Cost Computation</u>

Before proceeding with the computation of total discounted cost, some adjustment must be made to the year one and year six costs to reflect the phase in and phase out of the system. The best approach is to compute separate costs for these two periods. It is assumed that production will be evenly spaced throughout the year 1995 and that, likewise, the retirement of the system will be evenly spread throughout the year 2000 and that costs are proportional to the number of active systems. Thus, there will be an average of 36 systems in operation during these two years. The analysis was reaccomplished with 36 rather than 72 aircraft with the results summarized in Table 27 . Variances used in computing Var[Y] are listed in Appendix I .

TABLE 27

Phase In/Phase Out Yearly Cost Summary

$million FY83

Aircraft	Element	E[X]	SD[X]
XF-1	X_1	15.271	0.158
	X_2	14.624	0.148
	X_3	9.181	0.001
	X_4	3.984	0.162
	X_5	0.367	0.004
XF-2	X_1	15.622	0.151
	X_2	17.594	0.103
	X_3	8.839	0.001
	X_4	6.526	0.121
	X_5	0.284	0.002

For the XF-1,

E[Y] = 61.083
Var[Y] = 6.1342EE10

For the XF-2,

E[Y] = 66.557
Var[Y] = 4.1945EE10

Assuming costs are independent[21] from year to year, the expected value and variance of the present value of total operating and support costs are computed using equations (40) and (41) respectively. With a discount rate of ten

[21] This may or may not be a valid assumption. Additional research needs to be done in this area. As an alternative to the assumption of independence, complete dependence could be assumed. The two results, independence and complete dependence, could then be used to put upper and lower bounds on the cost.

percent, application of these equations leads to the results shown in Table 28 . It is important to note that the analysis is as of 1983 in 1983 dollars. Thus, the first year discounted is actually year thirteen and the last year eighteen.

TABLE 28

Discounted Total O&S Costs

$millions FY83

	XF-1	XF-2
E[PV(TOS)]	141.159	153.481
SD[PV(TOS)]	0.184	0.151
Note: Var[PV(TOS)]	3.3991EE10	2.2821EE10

4.5.3 Presenting the Analysis

Next an analysis and presentation of results is prepared. Although rather obvious from Table 28 , a mean-variance was prepared as shown in Figure 17 . From this plot, the decision maker's choice is not clear. The XF-1 has the lower expected value, but the higher variance. Therefore, further study is in order.

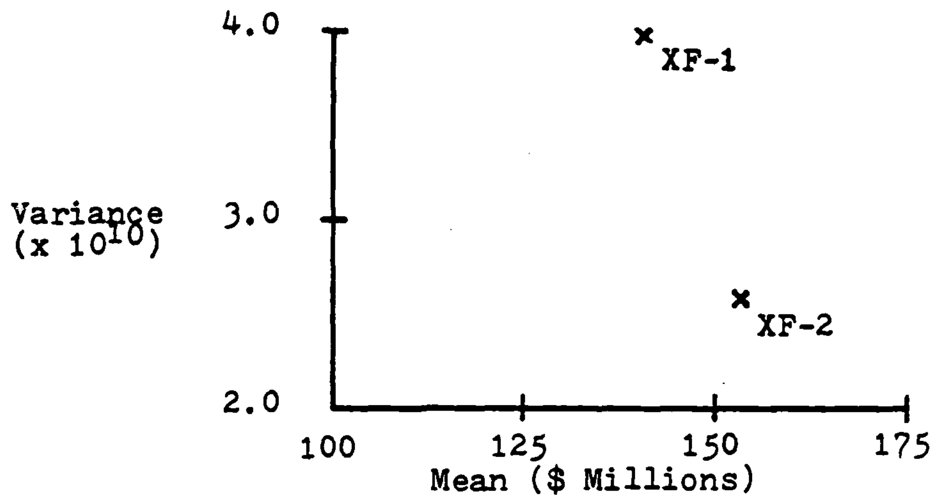


Figure 17: FoF Mean-Variance Plot

The computation of tolerance intervals is the next step. Using equation (101) and Table 11 which assume a normal population and infinite sample size, tolerance limits were computed such that at least ninety-five percent of the population was included with a point nine five (.95) probability level. These results were, for the XF-1,

(140.800,141.520)

and for the XF-2,

(153.190,153.780)

The choice is now clear. The tolerance intervals do not overlap and are, in fact, quite far apart. The XF-1 does in-

deed cost less to operate and support. The reader must however bear two things in mind. First, although the variance of both systems was quite large, it is the square root of the variance that is used in computing the tolerance intervals and it is rather small by comparison. The second point is that this estimate is for O&S costs only. Development, acquisition, and, perhaps, disposal costs must be considered in the life cycle cost evaluation. Development and acquisition costs could swing the pendulum in favor of the XF-2.

The reader is reminded that this analysis is valid only under the assumptions used in its preparation. The first and foremost assumption is the fixed FoF scenario. This assumption eliminates environmental uncertainty. Other assumptions include:

1. Use of the additive and multiplicative models
2. Use of the beta, rectangular, and Poisson distributions
3. Use of the normal distribution in computing yearly O&S and total discounted O&S cost
4. Appropriateness of the CER's
5. Adequacy of the Taylor series approximation
6. Independent cost from year to year

At this point, the risk analysis could be concluded, but the analyst still has some unanswered questions. These will be answered in performing an indepth risk analysis on selected cost elements.

4.6 INDEPTH RISK ANALYSIS

After performing the preliminary risk analysis on the cost drivers, an indepth detailed analysis should be accomplished where warranted. Such an analysis gives greater insight into the problem at hand. In this section, transforms will be used to examine the fuel and engine spares CER's and the depot maintenance analogy. The reader is reminded that the probability distributions used are for illustrative purposes and only the XF-1 will be used. Analysis of the XF-2 would be analogous to that of the XF-1.

4.6.1 Fuel

The fuel CER, equation (107), models the product of a constant and two random variables calling for application of the Mellin transform. Assuming that flying hours (F_1), fuel cost (F_2), and fuel consumption (F_3) are all triangularly distributed, the transform of the Mellin convolution of the fuel CER is

$$M[g(Z)] = C^{s-1} M[f(F_1)]\, M[f(F_2)]\, M[f(F_3)] \qquad (126)$$

In the case of the XF-1, the first factor on the right hand side is one. The Mellin transform of the triangular distribution is contained in Appendix D and for the variable F appears as

$$\begin{aligned} M[f(F)] = {} & C_1[(360^{s+1} - 336^{s+1})/(s+1) \\ & - ((336)(360^{s} - 336^{s+1})/s] \\ & + C_2[(408^{s+1} - (408)360^{s})/s \end{aligned}$$

$$- (408^{s+1} - 360^{s+1})/(s+1)\,] \qquad (127)$$

where

$$C_1 = 2/((408-336)(360-336))$$

$$C_2 = 2/((408-336)(408-360))$$

Transforms for F_2 and F_3 are similar.

The transform of $g(Z)$ represented by the convolution in (126) can then be inverted either by table lookup or contour integration, neither of which is too appealing unless the transform is that of a well known and easily recognized distribution. If the transform can not be inverted, all is not lost. The moments of the distribution of Z can be easily obtained by applying equation (32) and, in so doing,

$$E[Z] = \$419{,}305 \qquad (128)$$

and

$$Var[Z] = 974{,}387{,}650 \qquad (129)$$

both of which compare favorably to the previous estimate. Assuming normality, these new estimates can be used to update and refine the preliminary analysis. The assumption of normality can be checked using the moment coefficient of skewness (the third moment about the mean divided by the standard deviation cubed), however, this value would vary according to the values of a, m, and b which are used. Detailed calculations are contained in Appendix J .

4.6.2 Engine Spares

The CER for spare engines, (88), is once again the product of random variables. However, it is not a simple product as in the case of fuel due to the exponents on the variables. In spite of the complexity, it can be handled rather easily by the Mellin transform. If

$$X = F^E \tag{130}$$

where F is a random variable raised to the E power, the Mellin transform of the distribution of X is

$$M[g(X)] = M[f(F)]|_{s=E_s-E+1} \tag{131}$$

That is, the transform of F raised to the E power is the transform of the distribution of F with s replaced by (Es-E+1). For example, in the case of the spare engine CER, if one assumes that F_5 is rectangularly distributed, then the Mellin transform of the distribution of

$$W = F_5^{0.7974} \tag{132}$$

is

$$\begin{aligned} M[g(W)] &= ((a+b)^s-a^s)/(sb)|_{s=.7974s-.7974+1} \\ &= (2.05^n - 1.95^n)/m \end{aligned} \tag{133}$$

where

$$n = .7974s+.2026$$

$$m = .07974s+.02026$$

Taking the moments in accordance with equation (32), it can be easily shown that

$$E[W] = 1.74 \tag{134}$$

$$Var[W] = .0004 \tag{135}$$

Assuming F is triangularly distributed and applying the above to that distribution, it can be shown that the Mellin transform of g(Z) in (115) is

$$M[g(Z)] = C^{s-1} M[f(F_4)]|_{s=-1.6850s+2.6850}$$

$$M[f(F_5)]|_{s=.7974s+.2026} \qquad (136)$$

As in the previous case for fuel, the transform can be inverted and/or the moments can be obtained. Doing the latter,

$$E[Z] = 305 \qquad (137)$$

$$Var[Z] = 8459 \qquad (138)$$

which shows some change over the values previously obtained in doing the preliminary risk analysis. Detailed calculations using transforms for this CER are in Appendix K .

4.6.3 Depot Maintenance

Indepth risk analysis of depot maintenance provides an opportunity to demonstrate the use of the Laplace transform. Assume that upon further discussion with the experts, the analyst discovers that the values obtained for L and H in the change applied to the baseline in equation (49) are really not absolute but are more like L_δ and H_δ discussed under the gamma distribution and that there is some very small possibility that cost may not exceed the base. Appropriately, the analyst assumes that change (Z) is gamma distributed and applying equations (93)-(98) to the data in Table 19 for the XF-1, the parameters of the gamma distribution are

$$\alpha = 37.9392 \tag{139}$$

$$\beta = 0.1627 \tag{140}$$

Using the Laplace transform of the gamma distribution listed in Table 10 and applying the shifting theorem [38:32] which says that

$$L[f(X-a)U(X-a)] = \exp[-as]\, L[F(X)] \tag{141}$$

where the unit step function assures positiveness of the transform, the Laplace transform of the model

$$X = W + Z \tag{142}$$

is

$$L[g(X)] = \exp[-12.272s]\,(1+.1627s)^{-37.9392} \tag{143}$$

Applying equation (22), the moments can be extracted by taking appropriate derivatives of $L[g(X)]$ with respect to s and then evaluating at $s=0$. Such computation yields

$$E[X] = \$18.4428 \text{ million} \tag{144}$$

$$Var[X] = 1.0744 \text{ million} \tag{145}$$

Given the earlier assumptions in the section, these figures compare favorably with those obtained in the preliminary risk analysis. Detailed computations for the XF-1 are in Appendix L.

4.7 SENSITIVITY ANALYSIS

The last step in the full, complete analysis is a sensitivity analysis. Traditionally, this is done by varying a certain parameter or factor and noting the resulting change in cost. If that parameter or factor is a random variable, then a distribution accompanies that parameter or factor as it changes. This calls for a repeat of steps one, two, and three in order to assess the full impact of the sensitivity.

4.8 CONCLUDING REMARKS

This chapter has demonstrated the feasibility of a risk analysis under a typically realistic cost estimation scenario using proven and accepted CER's and modeling techniques. The four step approach offers a logical and reasonable way to confront the risk inherent in any program. To quell the objections to risk analysis discussed previously, more information is put before the decision maker, but it is information that can contribute substantially to the decision. In the case of the PoF, the decision maker would feel quite comfortable in choosing the XF-1 if the decision were based on discounted O&S cost. All decision makers have an inherent appreciation for risk as they are faced with it each day in every decision. Therefore, it is felt that decision makers do not lack understanding in this area. As for those who contend that the high numbers are too high, the width of

the tolerance interval is controlled to a large degree by the standard deviation which is, of course, the square root of the variance. In the problem presented, the variance was quite large, but the standard deviation was quite small by comparison. Therefore, the size of the numbers should not be a deterrent, particularly when one does not know what the numbers are in the first place. Lastly, risk analysis does not unfavorably impact the credibility of the study but, to the contrary, assures the decision maker that a thorough and complete analysis was done.

Theoretically, it is possible to do the whole risk analysis using transforms. Practically speaking, this is easier said than done. Starting with basic CER's and progressing to yearly and discounted total O&S costs, the mathematical expressions grow at each step to the point that they are next to impossible to manipulate. The problem is compounded when mixed additive and multiplcative models are used, forcing the inversion and retransformation of the expressions involved. If one is only concerned with the moments of the convolved distributions, the Mellin transform is far easier to manipulate than the Laplace. This may force a preference to the multiplicative model. Lastly, the transforms can be used quite easily and effectively in extracting the moments necessary to apply the method of additive and multiplicative moments introduced previously.

As a final remark, careful attention should be given to the additive verses multiplicative model issue and its impact on variance. Most CER's were developed with a point estimate in mind and from that standpoint, assuming independence, it really does not matter which model is used. However, when dealing with risk, the additive model can be used to control and reduce the variance. This may be of some import to those who say the high numbers are too high. For example, in the fuel CER, flying hours, fuel cost, and fuel consumption rate were multiplied giving cost per aircraft per year. In the case of the XF-1, this resulted in an expected value and variance of 406,224 and 604,766,259 respectively. If the analyst believes that the fuel cost may vary from one flying hour to the next depending on such things as changes in flight profile and aircraft gross weight, then the additive model is more appropriate. Using the additive model, the expected value remains the same but the variance is reduced to 1,168,903 , a reduction of over 600 million. Proper appplication of risk analysis should lead to a whole new modeling approach and set of CER's which are more reflective of reality and more accurate in terms of the actual risk present.

Chapter V

SUMMARY, RECOMMENDATIONS, AND FUTURE STUDIES

Life cycle cost (LCC) is one of the most controversial concepts being discussed today within the Department of Defense. It is such an intuitively appealing concept that it finds its way into the lexicon of virtually everyone involved in acquisition decisions. Unfortunately, while the concept itself is simple, implementation has proven to be quite difficult. LCC cuts across traditional disciplines and functional areas. It demands data from information systems designed for other purposes. It requires new methods of analysis and revised business practices. Yet it holds forth great potential in managing the nation's defense resources.

This dissertation constitutes a critical examination of life cycle costing and, in particular, operating and support (O&S) costing. In that LCC activities are necessarily a part of a dynamic environment, this examination has been from three perspectives: methodology, modeling, and applications.

Two aspects of the dynamic environment give credence and true meaning to life cycle costing. These are risk and uncertainty. Without these, life cycle costing would be a meaningless exercise. Yet it is these two aspects that are the source of many problems associated with LCC. The LCC estimate is the focal point of life cycle costing and, with the inclusion of risk and uncertainty, it is a random variable. The analysis leading to the estimate must address these aspects through risk analysis in the case of risk and sensitivity analysis in the case of uncertainty. Although this requirement is well recognized, risk analysis, in particular, has received little attention. This paper addressed many of the fundamental issues related to risk and uncertainty in LCC estimates.

5.1 METHODOLOGY

The key to the acceptance of life cycle costing as a decision and management tool is credibility. Steps must be taken, both from an organizational and business practices point of view, to ensure that credibility. LCC must be the concern of everyone; in a word, it must be institutionalized.

As a step toward institutionalization, a mechanism to ensure that the benefits of life cycle costing are actually realized must be established. This may necessitate some basic reorganization, assignment of new responsibilities, and changes in management practices.

Program uncertainty is the leading contributor to LCC incredibility and is the leading reason why procurements go astray. The key to program uncertainty is program stability. Although instability can never be totally eliminated, the government as a whole must take steps to increase stability. Congress should lengthen its planning horizon and the Air Force must better define its requirements. Industry cooperation is also essential and the benefits of increased program stability will be reaped by all concerned.

If stability is the key to program uncertainty, then the program manager is the key to LCC management implementation. With proper motivation, the program manager can be an effective force in the management, control, and monitoring of life cycle costs. But the program manager alone is not enough!

Highly skilled and qualified analysts must be available to assist. These analysts must be supplied with the tools of their trade, data and modeling methods and techniques, in order to perform their function. The data must be accurate, complete, and readily available while modeling methods and techniques must be appropriate, consistent, and easily applied.

Most LCC estimates are presented as point or 'most likely' estimates with no indication as to the inherent risk contained in the estimate. This practice is dangerous and

can be misleading. Further, the complete spectrum of information regarding the estimate is not conveyed to decision makers. In the chapter on modeling, risk analysis as it relates to life cycle costing was fully explored.

5.2 MODELING

With the various cost factors and cost elements which constitute the basis for the LCC estimate recognized as random variables, risk analysis is a natural and necessary part of the study. Risk analysis should be directed at the cost drivers and falls into two broad categories: analytical and Monte Carlo simulation. This study concentrated on the analytical category and presented three methods: additive and multiplicative moments, sums and products of random vaiables, and transforms. Major emphasis was placed on the last two.

Seemingly overlooked in the literature, the sums and products of random variables is a convenient and easily implemented method of performing a risk analysis. Although sometimes requiring approximations, this method is applicable in both the independent and dependent cases and requires the least amount of information to be useful.

The method of transforms is more sophisticated mathematically than the sums and products of random variables, yet produces more information on the random variables of con-

cern. This method, as presented, is only applicable in the independent case. Two transforms, the Laplace for the additive model and the Mellin for the multiplicative model, were addressed.

In applying any of these methods to O&S costs, the general modeling structure must be considered. In computing O&S costs, factors are added and multiplied to arrive at cost elements. This is done primarily through the techniques of analogy and parametric costing. The cost elements are then added to arrive at a yearly cost. Yearly costs are then discounted and added to arrive at a total cost over the operational life of the system.

When considering Monte Carlo simulation, the method of additive and multiplicative moments, and transforms, the choice of probability distributions for the random variables concerned is quite important. Although there is a multitude to chose from, the normal, log normal, triangular, beta, rectangular, gamma, and Poisson are considered likely candidates.

After performing the risk analysis, its presentation to decision makers is crucial. It must be clear, accurate, precise, and easily understood. Various presentations include the mean-variance plot, tolerance intervals, cumulative distribution curves, and floating bar graphs. But modeling serves no real purpose in the absence of application.

5.3 APPLICATION

By means of a typical Air Force O&S cost scenario, it was shown that a risk analysis is not only possible but reasonable to perform using the methods and techniques discussed in this paper.

In performing such an analysis, the four step approach is recommended. These steps are:

1. Determine most likely point estimate
2. Perform preliminary risk analysis on cost drivers
3. Accomplish indepth risk analysis where warranted
4. Do sensitivity analysis

By following these steps, analysts and decision makers can be assured that a complete, thorough analysis was done and that a high level of reliance can be placed in the results.

5.4 RECOMMENDATIONS

Recommendations for improvement in the area of methodology appear in the chapter by that name. They include changes in business practices, reorganization of the acquisition process with emphasis on the program manager, improved data sources, and updated models.

In the area of modeling and risk analysis, increased use of the additive model would not only improve the model's approximation of reality but also portray a more accurate picture of the inherent risk. This recommendation is independent of the modeling method used.

Also, with the tools now at hand and the insight gained through this research, risk analysis should be a required part of every LCC and O&S cost estimate. To do otherwise would be a disservice to decision makers and a discredit to life cycle costing.

5.5 FUTURE STUDIES

No study is ever complete in and of itself. As questions are answered and problems solved, new questions and problems take their place. Such is certainly the case with this research.

Operating and support cost estimation in the dependent case deserves more attention. As a first step, the nature and extent of the dependency must be explored and determined. The Visibility and Management of Operating and Support Cost (VAMOSC) program may provide the necessary data. VAMOSC is, however, in its infancy and, at this point, may lack the historical depth needed. When more is known about the dependency, modeling methods can be devised and improved to capture it. This may be particularly true for transforms where joint distributions are necessary to model dependency.

With reference to probability distributions, effort is needed to determine the proper distributions for the various cost elements which comprise O&S cost. At present, Desmatics, Inc. is under Air Force contract for this study. When

the results are known, modeling methods can be devised to take advantage of this newfound information. In the case of transforms, predetermined relationships could be developed and used by analysts in the field thereby taking full advantage of this powerful modeling tool.

Lastly, risk analysis and its impact must be studied from the point of view of decision makers. In this research, those who work for and supply information to decision makers were interviewed. Once risk analysis becomes more accepted, decision makers should be interviewed to determine the type of presentation preferred and the extent to which risk analysis influences decisions.

The sky ahead is not without turbulance but with determination and perseverence, it will be navigated. The mission will continue as long as the Air Force requires new systems for the defense of the nation.

Appendix A

INTERVIEW ROSTER

Name	Organization
Maj Dev Devers	OSD/PA&E
Lt Col Bob Owens	HQ USAF/ACM
Lt Col Rich Wallace	HQ USAF/ACMC
Lt Col Gene Tattini	HQ USAF/LEYE
Lt Col Don Crawford	HQ AFSC/ACCE
Mr Frank Fong	HQ AFSC/ACCE
Mr John Rosso	HQ AFSC/ALPA
Mr Vern Menker	ASD/ACCL
Capt Arnie Douville	ASD/ACCL
Mr Mike Enright	ASD/YZA
Mr Ron Vorhis	ASD/YZPR
Mr Roger Steinlage	HQ AFLC/ACME
Maj Paul Reid	HQ AFLC/LC (VAMOSC)
Lt Col Larry Rice	AFALD/XRS
Mr Tom Parry	AFALD/XRSA
Mr John Huff	AFALD/XRSA
Maj Les Takamura	AFAFC/CWC
Dr Dick Taliaferro	AFIT/LSY
Mr Roy Wood	AFIT/LSY

Mr Virgil Rehg	AFIT/LSY
Capt Larry Emmelhainz	AFIT/LSY

Appendix B

GENERAL INTERVIEW QUESTIONS

The following general questions were asked of those interviewed:

1. What does your organization do with respect to life cycle costing (O&S costing)?
   a) What are the inputs to the organization? From where do they come?
   b) What are the outputs? To where do they go? For what are they used?
   c) From where and/or from whom do you get your direction and authority?
   d) What, if any, decisions relating to LCC are made by you or your organization?
2. As you see it, what are the major strengths of life cycle costing (O&S costing)?
   a) What are the major faults (problems) with life cycle costing (O&S costing)?
   b) What should be done to improve life cycle costing (O&S costing)?

c) Do you think life cycle costing has a credibility problem?

3. Risk and uncertainty are facts of life. How should risk and uncertainty be addressed in life cycle costing (O&S costing)?

   a) How can risk and uncertainty be reduced?

   b) How should information relating to risk and uncertainty be presented?

Appendix C

ORGANIZATIONAL SPECIFIC INTERVIEW QUESTIONS

OSD & AF CAIG

1. What is the authority (regulations, etc.) for life cycle costing?
2. What is required in preparing and presenting an estimate?
3. For what do you use LCC estimates?
4. Is LCC really used in making decisions?
5. How much do LCC estimates influence decisions?
6. How worthwhile is risk analysis?
7. How should risk analysis results be presented?
8. What can be done about real world uncertainty?
9. How can real world uncertainty be addressed in terms of risk analysis?
10. What is the magnitude of real world uncertainty verses cost estimating uncertainty?
11. Congress appropriates separate funds for acquisition and support. Do you see this as a problem? If so, what can be done about it?

12. Do decision makers understand random, stochastic processes?

13. What really drives acquisition decisions?

USAF/ACMC

1. How are inputs to AFR 173-13 prepared?

2. What are the numbers (means, modes, etc.) in AFR 173-13?

3. Can risk and uncertainty information be incorporated into AFR 173-13? If so, how should this information be presented?

4. What is the CORE methodology?

AFSC/ACCE & AFSC/ALPA

1. What is AFSC policy regarding LCC?

2. How do the product divisions differ with respect to LCC issues?

ASD/ACCL

1. How do you see requirements for risk analysis affecting program offices?

2. At the working level, is LCC policy and procedure clear and concise?

ASD Program Offices

1. Do program office personnel understand risk analysis?

2. How would risk analysis affect your work load?

3. At the working level, is LCC policy and procedure clear and concise?

AFLC/ACME

1. How is historic data collected, treated, and disseminated?

2. What are the problems with the maintenance data collection system?

3. How can risk information be included?

<u>AFLC/LO VAMOSC</u>

1. What is VAMOSC?

2. What does it offer?

3. How does it compare to other AFLC data collection systems?

<u>AFALD/XRS</u>

1. At the working level, is LCC policy and procedure clear and concise?

2. Are regulations and instructions adequate?

<u>AFAFC/CWC</u>

1. How is data for AFR 173-13 collected?

2. Can risk analysis information be determined from existing data?

<u>AFIT</u>

1. What do you teach in the areas of LCC and risk analysis?

2. Are students aware of the stochastic nature of LCC?

Appendix D

THE TRIANGULAR DISTRIBUTION

Although rather easily derived from basic definitions, the properties of the triangular distribution presented here are rather lengthy due to the bounds placed on x and lack of smoothness in the density function.

$$f(x) = \begin{cases} (2(x-a))/((b-a)(m-a)) & 0 \le a \le x \le m \\ (2(b-x))/((b-a)(b-m)) & m \le x \le b \end{cases}$$

Let $c_1 = 2/((b-a)(m-a))$, $c_2 = 2/((b-a)(b-m))$

$$E[x] = \int_{-\infty}^{\infty} xf(x)\ dx = c_1\int_a^m x(x-a)\ dx + c_2\int_m^b x(b-x)\ dx$$

$$= c_1[(2m^3-3am^2+a^3)/6] + c_2[(b^3-3bm^2+2m^3)/6]$$

Let $E[x) = \mu$

$$Var[x] = \int_{-\infty}^{\infty}(x-\mu)f(x)\ dx = c_1\int_a^m (x-\mu)(x-a)\ dx$$

$$+ c_2\int_m^b (x-\mu)(b-x)\ dx$$

$$= c_1[(2m^3+a^3-(3a+3\mu)m^2-3\mu a^2+6\mu am)/6]$$

$$+ c_2[(b^3+2m^3-(3b+3\mu)m^2-3\mu b^2+6\mu bm)/6]$$

$$L[f(x)] = \int_0^\infty e^{-sx} f(x)\, dx = c_1\int_a^m (x-a)\, e^{-sx} dx + c_2\int_m^b (b-x)\, e^{-sx} dx$$

$$= c_1[(e^{-sm}(-sm-1)+e^{-sa}(sa+1))/s^2 + (ae^{-sm}-ae^{-sa})/s]$$

$$+ c_2[(e^{-sm}(-sm-1)+e^{-sb}(sb+1))/s^2 + (be^{-sm}-be^{-sb})/s]$$

$$M[f(x)] = \int_0^\infty x^{s-1} f(x)\, dx = c_1\int_a^m x^{s-1}(x-a)\, dx$$

$$+ c_2\int_m^b x^{s-1}(b-x)\, dx$$

$$= c_1[(m^{s+1}-a^{s+1})/(s+1) - (am^s-a^{s+1})/s]$$

$$+ c_2[(b^{s+1}-bm^s)/s - (b^{s+1}-m^{s+1})/(s+1)]$$

Appendix E

MAINTENANCE PERSONNEL CALCULATIONS

Type: Analogy

Data: Tables 13 , 14 , and 15

$MP = \text{Base} + \text{Change} = W + Z$

$$E[MP] = W + E[Z] = 1728 + 35 = 1763$$

$$Var[MP] = Var[Z] = 35$$

Using the additive model where the number of terms in the sum is a random variable,

$$E[X] = E[MP]\,E[PAY] = (1763)\ (17513) = 30{,}785{,}419$$

$$Var[X] = E[MP]\,Var[PAY] + E[PAY]^2\,Var[MP]$$

$$= (1763)\ (23703700) + (17513)^2\ (35)$$

$$= 5.2524EE10$$

Appendix F

FUEL CALCULATIONS

Type: Factor Based CER

Source: LSC Model

Data: Table 17

$Z = CF_1F_2F_3$

$$E[Z] = CF_1F_2F_3 | P_\mu$$

$$Var[Z] = (\partial Z/\partial F_1 | P_\mu)^2 \sigma^2_{F_1} + (\partial Z/\partial F_2 | P_\mu)^2 \sigma^2_{F_2} + (\partial Z/\partial F_3 | P_\mu)^2 \sigma^2_{F_3}$$

where

$$\partial Z/\partial F_1 = CF_2F_3$$

$$\partial Z/\partial F_2 = CF_1F_3$$

$$\partial Z/\partial F_3 = CF_1F_2$$

$$E[Z] = (1)\ (364)\ (2.00)\ (558) = 406{,}224$$

$$Var(Z) = (1245456)\ (144) + (4.1254EE10)\ (.000C) + (529984)\ (756)$$

$$= 604{,}766{,}259$$

Using the additive model for 72 aircraft,

$$E[X] = 29{,}248{,}128$$

$$Var[X] = 4.3543EE10$$

Appendix G

ENGINE SPARES CALCULATIONS

Type: Regression Based CER

Source: MLCCM

Data: Table 21

$$Z = CF_4^{-1.6850} F_5^{0.7974}$$

where

$$C = (1.7031 \times 10^6)\ (\text{ACTTYPE})$$

$$E[Z] = CF_4^{-1.6850} F_5^{0.7974} |_{P_\mu}$$

$$Var[Z] = (\partial Z/\partial F_4 |_{P_\mu})^2 \sigma^2_{F_5} + (\partial Z/\partial F_5 |_{P_\mu})^2 \sigma^2_{F_4}$$

where

$$\partial Z/\partial F_4 = -1.6850\ C\ F_4^{-2.6850}\ F_5^{0.7974}$$

$$\partial Z/\partial F_5 = 0.7974\ C\ F_4^{-1.6850}\ F_5^{-0.2026}$$

$$E[Z] = C\ (233)^{-1.6850} (2.0)^{0.7974} = 304$$

$$Var[Z] = (4.8202)\ (1111) + (14651)\ (.0008)$$

$$= 5367$$

Using the multiplicative model to compute cost per aircraft per year,

$$E[X^*] = (364)\ (5367) = 110{,}656$$

$$Var[X^*] = (304)^2\ (144) + (364)^2\ (5367) + (144)\ (5367)$$

$$= 7.2519EE08$$

Using the additive model to compute the cost for 72 aircraft,

$E[X] = 7{,}967{,}232$

$Var[X] = 5.2214EE10$

Appendix H

AVIONICS SPARES CALCULATIONS

Type: Regression Based CER

Source: MLCCM

Data: Table 23

$Z = C\ F_4^{0.8545} F_1^{-0.1169} F_5^{0.7979}$

where

$$C = 2.2956EE\text{-}3$$

$$E[Z] = C\ F_4^{0.8545} F_1^{-0.1169} F_5^{0.7979} |_{P_\mu}$$

$$Var[Z] = (\partial Z/\partial F_4 |_{P_\mu})^2 \sigma^2_{F_4} + (\partial Z/\partial F_1 |_{P_\mu})^2 \sigma^2_{F_1} + (\partial Z/\partial F_5 |_{P_\mu})^2 \sigma^2_{F_5}$$

where

$$\partial Z/\partial F_4 = (.8545)\ C\ F_4^{-0.1455} F_1^{-0.1169} F_5^{0.7979}$$

$$\partial Z/\partial F_1 = (-.1169)\ C\ F_4^{0.8545} F_1^{-1.1169} F_5^{0.7979}$$

$$\partial Z/\partial F_5 = (.7979)\ C\ F_4^{0.8545} F_1^{-0.1169} F_5^{-0.2021}$$

$$E[Z] = 28.0$$

$$Var[Z] = (.0826)\ (2.08) + (.0001)\ (144) + (.0001)\ (20833) = 2.2695$$

Using the multiplicative model, cost per aircraft per year,

$X^* = F_1\ Z$

$$E[X^*] = E[F_1]\ E[Z] = 10,192$$

$$\begin{aligned}
Var[X^*] &= E[Z]^2 Var[F_1] + E[F_1]^2 Var[Z] + Var[F_1]\,Var[Z] \\
&= (28)^2\ (144) + (364)^2\ (2.2695) \\
&\quad + (144)\ (2.2695) \\
&= 413{,}922
\end{aligned}$$

Using the additive model to compute the cost for 72 aircraft,

$$E[X] = 733{,}834$$

$$Var[X] = 29{,}802{,}384$$

Appendix I

COST ELEMENT VARIANCE SUMMARY

Risk Analysis Yearly Variance

Aircraft	Element	Var[X]
XF-1	X_1	5.2524EE10
	X_2	4.3543EE10
	X_3	8.4030EE05
	X_4	5.2214EE10
	X_5	2.9802EE07
XF-2	X_1	4.5354EE10
	X_2	2.1035EE10
	X_3	2.5840EE05
	X_4	2.9350EE10
	X_5	8.7082EE06

Phase In/Phase Out Yearly Variance

Aircraft	Element	Var[X]
XF-1	X_1	2.4963EE10
	X_2	2.1772EE10
	X_3	6.4600EE05
	X_4	2.6107EE10
	X_5	1.4901EE07
XF-2	X_1	2.2677EE10
	X_2	1.0518EE10
	X_3	6.4600EE05
	X_4	1.4675EE10
	X_5	4.3540EE06

Appendix J

MELLIN TRANSFORM EXAMPLE - FUEL CER

$$M[g(Z)] = C^{s-1} M[f(F_1)]\, M[f(F_2)]\, M[f(F_3)]$$

$$M[f(x)] = c_1[(m^{s+1}-a^{s+1})/(s+1) - (am^s-a^{s+1})/s]$$
$$+ c_2[(b^{s+1}-bm^s)/s - (b^{s+1}-m^{s+1})/(s+1)] \text{ br}$$
$$= c_1V_1 + c_2V_2$$

where

$$c_1 = 2/((b-a)(m-a)), \quad c_2 = 2/((b-a)(b-m))$$

For F_1,

$c_1 = .0011574074 \quad c_2 = .0005787037$

If s=2,

$V_1 = 101375.9999 \quad V_2 = 433152$

If s=3,

$V_1 = 35693568 \quad V_2 = 163012608$

then,

$E[F_1] = 367.9999$

$E[F_1^2] = 135648$

$Var[F_1] = E[F_1^2] - E[F_1]^2 = 224.0736$

For F_2,

$c_1 = 266.6666 \quad c_2 = 133.3333$

If s=2,

$V_1 = .0024791667 \quad V_2 = .01016666$

If s=3,

$V_1 = .0049171875 \quad V_2 = .020675$

then,

$E[F_2] = 2.0167$

$E[F_2^2] = 4.067916665$

$Var[F_2] = .0009722209$

For F_3,

$c_1 = .0002020202 \quad c_2 = .0001154401$

If s=2,

$V_1 = 953999.9994 \quad V_2 = 3224812.499$

If s=3,

$V_1 = 505980000.1 \quad V_2 = 1889891719$

then,

$E[F_3] = 564.9999$

$E[F_3^2] = 320387.5$

$Var[F_3] = 1162.50117$

Then,

$E[Z] = 419,305.3301$

$E[Z^2] = 1.7679135EE11$

$Var[Z) = 974,387,650.9$

Appendix K

MELLIN TRANSFORM EXAMPLE - SPARE ENGINE CER

$Z = C\,F_4^{-1.6850}F_5^{0.7974}$

where

$C = 1.7031EE06$

$M[g(Z)] = C^{s-1}M[f(F_4)]_{s=-1.685s+2.6850}M[f(F_5)]_{s=.7974s+.2026}$

Assume F_4 is triangularly distributed,

$$M[f(F_4)] = c_1[(m^{s+1}-a^{s+1})/(s+1) - (am^s-a^{s+1})/s]$$
$$+ c_2[(b^{s+1}-bm^s)/s - (b^{s+1}-m^{s+1})/(s+1)]$$

where

$c_1 = 2/((b-a)(m-a)) = .00013333$

$c_2 = 2/((b-a)(b-m)) = .00008$

If s=2,

$$E[F_4] = c_1(2.096291903 - 1.715955431)$$
$$+ c_2(3.266001895 - 2.610765295)$$
$$= .0001031305$$

If s=3,

$$E[F_4] = c_1(.0003248294 - .0002720343)$$
$$+ c_2(.0002552306 - .0001985804)$$
$$= .0000000116$$

Assume F_5 is rectangularly distributed,

$M[f(F_5)] = ((a+b)^s - a^s)/(sb)$

where

$$a = 1.95, \quad b = .1$$

$$E[F_5] = 1.73793692$$

$$E[F_5^2] = 3.020824875$$

Then,

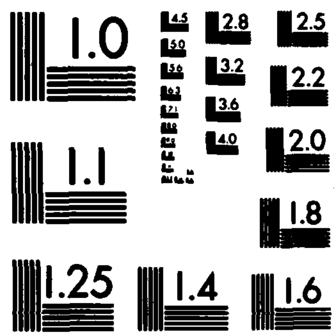
$$E[Z] = 305.2539423$$

$$E[Z^2] = 101639.808$$

$$Var[Z] = 8459.838678$$

AD-A133 023 LIFE CYCLE COSTING IN A DYNAMIC ENVIRONMENT(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH J A LONG 1983 AFIT/CI/NR-83-26D

UNCLASSIFIED F/G 5/1 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Appendix L

LAPLACE TRANSFORM EXAMPLE - DEPOT MAINTENANCE

$X = W + Z$

Assume Z is gamma distributed

$$\mu = (L_\delta + .95M + H_\delta)/2.95 = 6.1727$$

$$\sigma^2 = ((H_\delta - L_\delta)/3.25)^2 = 2.8639$$

$$\beta = \mu - M = .1627$$

If $k=0$,

$$\alpha = \mu/\beta = 37.9392$$

$$L[h(Z)] = (1 + .1627s)^{-37.9392}$$

and

$$L[g(X)] = \exp[-12.272s]\,(1 + .1627s)^{-37.9392}$$

$$E[X] = 18.4428$$

$$E[X^2] = 341.2113$$

$$Var[X] = 1.0744$$

BIBLIOGRAPHY

1. Aircraft Operating and Support Cost Development Guide. Washington, D.C.: Office of the Secretary of Defense, Cost Analysis Improvement Group, 15 April 1980.

2. AF Regulation 173-13. USAF Cost and Planning Factors. Washington, D.C.: Department of the Air Force, Headquarters US Air Force, 1 February 1982.

3. AF Regulation 178-1. Economic Analysis and Program Evaluation for Resource Management. Washington, D.C.: Department of the Air Force, Headquarters US Air Force, 28 December 1973.

4. AF Regulation 400-31, Volume I. Visibility and Management of Operating and Support Cost Program Policy and Procedures. Washington, D.C.: Department of the Air Force, Headquarters US Air Force, 30 September 1982.

5. AF Regulation 400-31, Volume II. Visibility and Management of Operating and Support Cost Program Weapon Systems Support Costs (WSSC). Washington, D.C.: Department of the Air Force, Headquarters US Air Force, 24 August 1982.

6. AF Regulation 800-11. Life Cycle Cost Management Program. Washington, D.C.: Department of the Air Force, Headquarters US Air Force, 22 February 1978.

7. Arrow, Kenneth J. Essays in the Theory of Risk-bearing. Chicago: Markham Publishing Company, 1971.

8. Babiarz, Captain Anthony S. and Captain Peter W. Giedras. "A Model to Predict Final Cost Growth in a Weapon System Development Program," Master's thesis, Air Force Institute of Technology, August 1975 (AD A016040).

9. Baker, William R. "Life Cycle Costing." Plant Engineering 32 (May 1978): 171-176.

10. Barry, Austin. Errors in Practical Measurement in Science, Engineering, and Technology. New York, New York: John Wiley & Sons, Inc., 1978.

11. Berard, Andrew H. Life Cycle Cost Management, Methodology, and Case Studies. Ft. Belvoir, Virginia: Defense Systems Management College, 1977 (AD-A052388).

12. Blanchard, Benjamin S. Design and Manage to Life Cycle Cost. Portland, Oregon: M/A Press, 1978.

13. Borch, Karl Henrik. The Economics of Uncertainty. Princeton, New Jersey: Princeton University Press, 1968.

14. Brown, Rex V. Research and the Credibility of Estimates. Homewood, Illinois: Richard D. Irwin, Inc., 1969.

15. Brown, Robert J. and Rudolph R. Yanuck. Life Cycle Costing: A Practical Guide for Energy Managers. Atlanta: The Fairmont Press, Inc., 1980.

16. Buckingham, Major General Charles E. "A Look at Acquisition Logistics." Air University Review. (May-June 1975): 35-40.

17. Canada, John R. Intermadiate Economic Analysis for Management and Engineering. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1971.

18. Canada, John R. and John A. White, Jr. Capital Investment Decision Analysis for Management and Engineering. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1980.

19. Churchman, C. West, Leonard Auerbach, and Simcha Sadan. Thinking for Decisions Deductive Quantitative Methods. Chicago, Illinois: Science Research Associates, Inc., 1975.

20. Cira, Captain Anthony T. and Captain Kenneth R. Jennings. "Life Cycle Costing: A Working Level Approach," Master's thesis, Air Force Institute of Technology, June 1981 (AD-A103256).

21. Coon, Helen. "Note on William A] Donaldson's 'The Estimation of the Mean and Variance of a 'PERT' Activity Time'." Operations Research 13 (May-June 1965): 386-387.

22. Cost of Ownership Handbook. Kirtland AFB, New Mexico: Director of Analysis, Air Force Test and Evaluation Center, May 1976.

23. Cost Uncertainty/Management Reserve Analysis. Eglin AFB, Florida: Coost Management and Technology Division, Directorate of Cost Analysis, Deputy for Comptroller, Armament Division, January 1982.

24. Cozzolino, John M. "A New Method for Risk Analysis." Sloan Management Review 20 (Spring 1979): 53-66.

25. Cramer, H. Mathematical Methods of Statistics. Princeton, New Jersey: Princeton University Press, 1946.

26. "Design to Cost and Life Cycle Cost," Amsterdam, Netherlands: AGARD Conference Proceedings No. 289, May 1980 (AD A090098).

27. Dienemann, Paul F. Estimating Cost Uncertainty Using Monte Carlo Techniques. Santa Monica, California: The Rand Corporation, RM-4854-PR, January 1966 (AD-629082).

28. Doetsch, Gustav. Guide to the Application of Laplace Transforms. London, England: D. VanNostrand Company Ltd., 1961.

29. Donahue, James D. Products and Quotients of Random Variables and Their Applications. Denver, Colorado: The Martin Company, July 1974 (AD-603667).

30. Donaldson, William A. "The Estimation of the Mean and Variance of a 'PERT' Activity Time." Operations Research 13 (May-June 1965): 382-385.

31. Faber, Conrad W. Probabalistic Estimates with Limited Data. St Louis, Missouri: US Army Aviation Research & Development Command, Technical Memorandum TM 81-F-1, November 1980 (AD A093969).

32. Fallon, Richard. Subjective Assessment of Uncewrtainty. Santa Monica, California: The Rand Corporation, P-5581, 1976 (AD-A026669).

33. Fisher, G. H. The Nature of Uncertainty. Santa Monica, California: The Rand Corporation, P-5133, November 1973 (AD-A044831).

34. Fisher, Gene H. Cost Considerations in Systems Analysis. Santa Monica, California: The Rand Corporation, R-490-ASD, December 1970.

35. Fleischer, Gerald A. Risk and Uncertainty: Non-Deterministic Decision Making in Engineering. Norcross, Georgia: American Institute of Industrial Engineers, Inc., 1975.

36. Freund, Rudolf J. and Paul D. Minton. Regression Methods A Tool For Data Analysis. New York, New York: Marcel Dekker, Inc., 1979.

37. Giffin, Walter C. Introduction to Operations Engineering. Homewood, Illinois: Richard D. Irwin, Inc., 1971.

38. Giffin, Walter C. Transform Techniques for Probability Modeling. New York: Academic Press, 1975.

39. Gnedenko, B. V. The Theory of Probability. New York, New York: Chelsea Publishing Company, 1967.

40. Graves, Samuel B. "A Monte Carlo Risk Analysis of Life Cycle Cost Prediction," Master's thesis, Air Force Institute of Technology, 1975 (AD-A021677).

41. Hahn, Gerald J. and Samual S. Shapiro. Statistical Models in Engineering. New York: John Wiley & Sons, Inc., 1967.

42. Haight, Frank A. Handbook of the Poisson Distribution. New York, New York: John Wiley & Sons, Inc., 1967.

43. Haugen, Edward B. Probabalistic Approaches to Design. New York, New York: John Wiley & Sons, Inc., 1968.

44. Heinze, David Charles. Statistical decision Analysis for Management. Columbus, Ohio: Grid, Inc., 1973.

45. Hertz, David B. "Risk Analysis in Capital Investment." Harvard Business Review. 57 (September-October 1979): 169-181.

46. Hillier, Frederick S. The Evaluation of Risky Interrelated Investments. Amsterdam: North-Holland Publishing Company, 1969.

47. Hillier, Frederick S. and Gerald J. Lieberman. Introduction to Operatopns Research. San Francisco: Holden-Day, Inc., 1980.

48. Holt, Charles A. Jr. and Robert W. Shore. Bayesian Analysis in Economic Theory and Time Series Analysis. Amsterdam: North-Holland Publishing Company, 1980.

49. Hutton, R. Bruce and William L. Wilkie. "Life Cycle Cost: A New Form of Consumer Information." Journal of Consumer Research 6 (March 1980): 349-360.

50. Interviewer's Manual Revised Edition. Ann Arbor, Michigan: The University of Michigan, Survey Research Center, Institute for Social Research, 1976.

51. Isaacson, Walter. "The Winds of Reform." Time 121 (March 7, 1983): 12-30.

52. Jackson, P. S., R. W. Hockenbury, and M. L. Yeater. "Uncertainty Analysis of System Reliability and Availability Assessment." Nuclear Engineering and Design 68 (1981): 5-29.

53. Jewell, William S. Planning and Control under Risk. Berkley, California: Operations Research Center, University of California, Berkley, August 1976 (AD A037013).

54. Joint AFSC/AFLC Life Cycle Cost Management Implementation Techniques Pamphlet. (Draft publication.) Andrews AFB, Maryland: AFSC/AFLC LCC Steering Group, July 1981.

55. Kahneman, Daniel and Amos Tversky. "Prospect Theory: An Analysis of Decision Under Risk." Econometrics 47 (March 1979): 263-291.

56. Kendall, Maurice G. and Alan Stuart. The Advanced Theory of Statistics. New York: Hafner Publishing Company, 1967.

57. Kernan, John E. and Lavern J. Menker. Life Cycle Cost Procurement Guide. Wright-Patterson AFB, Ohio: Joint AFSC/AFLC Commanders' Working Group on Life Cycle Cost, ASD/ACL, July 1976.

58. Lamar, William E. Technical Evaluation Report on Design to Cost and Life Cycle Cost. AGARD Advisory Report No. 165, 16 July 1981 (AD A101447).

59. Laning, J. Halcombe, Jr. and Richard H. Battin. Random Processes in Automatic Control. New York, New York: McGraw-Hill Book Company, Inc., 1956.

60. Large, Joseph P. Bias in Initial Cost Estimates: How Low Estimates Can Increase the Cost of Acquiring Weapon Systems. Santa Monica, California: The Rand Corporation, R-1467-PA&E, July 1974 (AD-787395).

61. Large, Joseph P., Harry G. Campbell, and David Cates. Parametric Equations for Estimating Aircraft Airframe Costs. Snata Monica, California: The Rand Corporation, R-1693-1-PA&E, February 1976 (AD-A022086).

62. Lenox, Hamilton T. "Risk Assessments," Master's thesis, Air Force Institute of Technology, June 1973 (AD-767871).

63. Life Cycle Cost Guide for Major Weapon Systems. Naval Material Command, November 1977 (AD A082273).

64. Lochry, Colonel Robert R. Final Report of the USAF Academy Risk Analysis Study Team. Colorodo Springs, Colorodo: United States Air Force Academy, 1 August 1971 (AD-729223).

65. Logistics Support Cost Model User's Handbook Version 1.1 . Wright-Patterson AFB, Ohio: Headquarters, Air Force Acquisition Logistics Division, January 1979.

66. Lukacs, Eugene. Characteristic Functions. New York, New York: Hafner Publishing Company, 1960.

67. Marks, Kenneth E., H. G. Massey, and Brent D. Bradley. Life Cycle Cost Estimation for USAF Aircraft Systems: An Appraisal of Cost Element Structures and Estimating Methodologies. Santa Monica, California: The Rand Corporation, WN-9925-AF, September 1977.

68. Martin, J. J. Bayesian Decision Problems and Markov Chains. Huntington, New York: Robert E. Krieger Publishing Company, 1975.

69. May, Major Thomas E. Operating and Support Cost Estimating. Wright-Patterson AFB, Ohio: Life Cycle Cost Management Division, Directorate of Cost Analysis, Comptroller, Aeronautical Systems Division, May 1982.

70. McClure, Lucille. Life Cycle Costing. Orlando, Florida: Martin Marietta Aerospace, October 1976 (AD A030554).

71. McNichols, Gerald R. "On the Treatment of Uncertainty in Parametric Costing," PhD dissertation, The George Washington University, 1976.

72. Mendenhall, William and Richard L. Scheaffer. Mathematical Statistics with Applications. North Scituate, Massachusetts: Duxbury Press, 1973.

73. Menker, Lavern J. Life Cycle Cost Analysis Guide. Wright-Patterson AFB, Ohio: Joint AFSC/AFLC Commanders' Working Group on Life Cycle Cost, ASD/ACL, November 1975.

74. Mitchell, Terence R. Uncertainty and Decision Making. Seattle, Washington: Department of Psychology, University of Washington, 1 September 1979 (AD-A074072).

75. Modular Life Cycle Cost Model for Advanced Aircraft Systems Phase III, Volume V, Operations and Support Phase Expansion. Bethpage, New York: Grumman Aerospace Corporation, AFFDL-TR-78-40, Volume V, January 1980.

76. Moore, Peter G. Risk in Business Decisions. London: Longman Group Ltd., 1972.

77. Morris, Peter A. "Combining Expert Judgements: A Bayesian Approach." Management Science 23 (March 1977): 679-693.

78. Morse, H. Stephen. A Comparison of Risk Assessment Methodologies. McLean, Virginia: System Development Corporation, 30 June 1980 (AD A089346).

79. Murphy, Edward L., Jr. Statistical Methods of Measuring the Uncertainty of Cost Estimates. Redstone Arsenal, Alabama: Cost Analysis Division, Comtroller and Director of Programs, US Army Missile Command, February 1970 (AD-718862).

80. Naylor, Thomas H., Joseph L. Balintfy, Donald S. Burdick, and Kong Chu. Computer Simulation Techniques. New York: John Wiley & Sons, Inc., 1966.

81. Neter, John and William Wasserman. Applied Linear Statistical Models. Homewood, Illinois: Richard D. Irwin, Inc., 1974.

82. Operating and Support Cost Development Guide for Aircraft Systems. Washington, D.C.: Cost Analysis Improvement Group, May 1974 (AD A001747).

83. Ostle, Bernard and Richard W. Mensing. Statistics in Research. Ames, Iowa: The Iowa State University Press, 1975.

84. Parratt, Lyman G. Probability and Experimental Errors in Science. New York, New York: John Wiley & Sons, Inc., 1961.

85. Payne, Stanley L. The Art of Asking Questions. Princeton, New Jersey: Princeton University Press, 1951.

86. Perry C. and I. D. Greig. "Estimating the Mean and Variance of Subjective Distributions in PERT and Decision Analysis." Management Science 21 (August 1975): 1477-1480.

87. Poindexter, Lt Colonel Eugene O. "Cost for Decision Making: A Synopsis." Armed Forces Comptroller (January 1976): 16-23.

88. Pratt, John W., Howard Raiffa, and Robert Schlaifer. Introduction to Statistical Decision Theory. New York: McGraw-Hill Book Company, 1965.

89. Proceedings of Management of Risk and Uncertainty in the Acquisition of Major Programs. Colorodo Springs, Colorodo: US Air Force Academy, February 1981.

90. Quade, E. S. Analysis for Military Decisions. Santa Monica, California: The Rand Corporation, R-387-PR, November 1964.

91. Recktenwalt, Thomas J. The Air Force Visibility and Management of Operating and Support Costs A Practical Approach to O&S Costing. Arlington, Virginia: paper presented to 17th Annual DOD Cost Analysis Symposium, 12-15 September 1982.

92. Sampson, Allan R. and Robert L. Smith. Assessing Risks Through the Determination of Rare Event Probabilities. Pittsburg, Pennsylvania: University of Pittsburg, Department of Mathematics and Statistics, July 1980 (AD A092192).

93. Schaefer, Donald F., Frank J. Husic, and Michael F. Gutowski. A Monte Carlo Simulation Approach to Cost-Uncertainty Analysis. McLean, Virginia: Research Analysis Corporation, March 1969 (AD-850054).

94. Schumacher, William J. Life Cycle Costing and the Effect of Ownership Costs. Ft Belvoir, Virginia: Defense Systems Management School, November 1975 (AD A027288).

95. Seldon, M. Robert. Life Cycle Costing: A Better Method of Government Procurement. Boulder, Colorodo: Westview Press, 1979.

96. Selvidge, Judith. Methodology for the Retrospective Evaluation of Decision Analysis. McLean, Virginia: Decisions and Designs, Inc., September 1976 (AD A035856).

97. Springer, M. D. The Algebra of Random Variables. New York, New York: John Wiley & Sons, 1979.

98. Sullivan, William G. and W. Wayne Claycombe. Fundamentals of Forecasting. Reston, Virginia: Reston Publishing Company, Inc., 1977.

99. Summers, Robert. Cost Estimates as Predictors of Actual Weapon Costs: A Study of Major Hardware Articles. Santa Monica, California: The Rand Corporation, RM-3061-PR, March 1965 (AD-612723).

100. Thompson, Gerald E. Statistics for Decisions An Elementary Introduction. Boston: Little, Brown and Company, 1972.

101. US Army Total Risk Assessing Cost Estimates (TRACE) Guidelines. Hopewell, Virginia: John M. Cockerham & Associates, Inc., 31 December 1976 (AD-A036327).

102. User's Guide for the Timeshare Computer Program Implementation of the Cost Oriented Resource Estimating (CORE) Model. Wright-Patterson AFB, Ohio: Life Cycle Cost Management Division, Directorate of Cost Analysis, Comptroller, Aeronautical Systems Division, 30 September 1981.

103. Wilks, Samuel S. Mathematical Statistics. New York: John Wiley & Sons, Inc., 1962.

104. Worm, George H. Application of Risk Analysis in the Acquisition of Major Weapon Systems. Clemson, South Carolina: Clemson University, 1 August 1980.

105. Worm, George H. Applied Risk Analysis with Dependance Among Cost Components. Clemson, South Carolina: Clemson University, November 1981.

END

FILMED

10-83

DTIC